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CLOSURE, CONTRAST, AND ANSWER

ABSTRACT. How should the contrastivist formulate closure? That is, given that knowledge is a ternary contrastive state $Kspq$ (s knows that p rather than q), how does this state extend under entailment? In what follows, I will identify adequacy conditions for closure, criticize the extant invariantist and contextualist closure schemas, and provide a contrastive schema based on the idea of extending answers. I will conclude that *only the contrastivist* can adequately formulate closure.

1. CONTRASTIVISM AND CLOSURE

There are two main assumptions I will be making that should be flagged from the start. First, I will be assuming a contrastive theory of knowledge.¹ That is, I will be assuming that knowledge is a ternary contrastive state $Kspq$ (where s is the subject, p is the proposition known, and q is the contrast proposition), whose evidential component $Espq$ is the elimination of the q -worlds. This assumption will not be directly defended here. Though what follows may be considered an indirect defense, insofar as contrastivism proves needed for an adequate formulation of closure.

Second, I will be assuming that some form of closure holds. After all, (i) the idea that knowledge extends under entailment is intuitively compelling (Feldman, 1995); (ii) mathematical knowledge, as based on proof, presupposes such extension (Williamson, 2000); and (iii) denying closure allows abominable conjunctions such as “I do not know that I am not a (handless) brain-in-a-vat, but I know that I have hands” (DeRose, 1995). This assumption will not be further defended here.² Though what follows may be considered an indirect defense, insofar as contrastive closure proves safe from objectionable consequences.

Putting this together, I will be assuming:

- (A1) *Contrastivism*: The knowledge state is $Kspq$, where q is a contrast proposition, and whose evidential component $Espq$ is the elimination of the q -worlds.
- (A2) *Closure*: $Kspq$ satisfies some closure schema.

So the question becomes, *what closure schema holds for the contrastive state $Kspq$?*

2. BINARY CLOSURE

It will prove useful to begin with closure schemata for binary Ksp . This will help identify adequacy conditions, illustrate some useful maneuvers, and reveal problems with binarity.

The simplest binary closure schema runs:

$$(1) \quad (Ksp_1 \ \& \ (p_1 \rightarrow p_2)) \rightarrow Ksp_2$$

In words: if s knows that p_1 and p_1 entails p_2 , then s knows that p_2 . In short, knowledge is closed under entailment.

What can be said for 1 is that it preserves the intuitions behind closure. That is, it provides a plausible rendering of how knowledge extends under entailment, explains how proof extends knowledge, and blocks abominable conjunctions. These make 1 a worthy candidate to be a closure schema. This yields three adequacy conditions on closure:

- (C1) An adequate closure schema should provide a plausible rendering of how knowledge extends under entailment.
- (C2) An adequate closure schema should explain how proof extends knowledge.
- (C3) An adequate closure schema should block abominable conjunctions. To fail C1–C3 is to be unsuited for the role of closure.

What can be said against 1 is that it faces counterexamples. First, s might fail to see some consequences of what he knows. We are not logically omniscient. Second, s might see some consequence but still fail to believe it. Third, s might

see and believe some consequence, but on the wrong basis. Perhaps s only forms his belief from reading tea leaves. Thus 1 is false. This yields three further adequacy conditions:

- (C4) An adequate closure schema should block lack-of-omniscience counterexamples.
- (C5) An adequate closure schema should block lack-of-belief counterexamples.
- (C6) An adequate closure schema should block irrational-basis counterexamples.

To fail C4–C6 is to be inapplicable to human knowledge.

An improved binary closure schema runs:

$$(2) \quad (\mathbf{K}sp_1 \ \& \ \mathbf{K}s(p_1 \rightarrow p_2)) \rightarrow \mathbf{K}sp_2$$

In words: if s knows that p_1 and s knows that p_1 entails p_2 , then s knows that p_2 . In short, knowledge is closed under *known* entailment.³

What can be said for 2 is that it satisfies C1–C4. That is, 2 preserves the virtues of 1 with respect to C1–C3.⁴ And by requiring that the entailment be known, 2 blocks lack-of-omniscience counterexamples, thus satisfying C4. But what can be said against 2 is that it fails C5–C6. That is, 2 remains subject to counterexample in cases where s fails to believe that p_2 , or s believes that p_2 on an irrational basis (perhaps by ignoring the proof in favor of trusting the tea leaves).

Perhaps the best binary closure schema revises the consequent of 1, replacing $\mathbf{K}sp_2$ with $\mathbf{E}sp_2$, where E is the evidential component of K:

$$(3) \quad (\mathbf{K}sp_1(p_1 \rightarrow p_2)) \rightarrow \mathbf{E}sp_2$$

That is, *position to know* is closed under entailment, where position to know is understood as satisfying the evidential component of knowledge. So if $\mathbf{E}sp_2$, then s has propositional justification for p_2 , and is perhaps only a belief-based-on-competent-deduction-while-retaining-knowledge-of- p_1 away from $\mathbf{K}sp_2$.⁵

What can be said for 3 is that it satisfies C1–C6. Essentially, 3 preserves the virtues of 1 with respect to C1–C3. And

3 satisfies C4–C6 because failures of omniscience, belief, and basing are no impediment to E. Thus the progression from 1 to 3 reveals how to surmount the omniscience, belief, and basing problems. The binary theorist might well rest here.

What can be said against 3 (and binary closure generally) is that it engenders skeptical paradox. To begin with, 3 is incompatible with A1. 3 is formulated for K_{sp} , not for the contrastive K_{spq} . Of course, the binary theorist will hardly flinch. But A1 is an assumption of the current discussion (§1).

Further, 3 precludes epistemic modesty. Surely we possess modest knowledge of the external world. For instance, Moore knows that he has hands. He need only look. And surely we also suffer modest ignorance of the external world. For instance, Moore does not know that he is not a brain-in-a-vat. He may look all he wants, but it won't help. But 3 forces our modest knowledge and modest ignorance into conflict, driving us to dogmatism or skepticism.⁶

Finally, 3 mistreats knowledge-*whether*. Intuitively, Moore knows whether he has hands or stumps. As to whether he has hands or stumps, he can tell by looking. This is an easy question. And intuitively, Moore does not know whether he has hands or vat-images of hands. As to whether he has hands or vat-images of hands, he can't tell – they'd look the same to him. This is an (impossibly) difficult question.

Yet 3, together with the standard binary view of knowledge-*whether*, forces these intuitive claims into conflict. The standard binary view of knowledge-*whether* reduces it to K_{sp} , where p is the true answer to the whether-question.⁷ So (given that Moore in fact has hands), his knowledge of whether he has hands or stumps *is* his knowledge that he has hands. This alleged knowledge would position Moore to know that he is not a brain-in-a-vat. This would position him to know whether he has hands or is a brain-in-a-vat, which of course he is in no position to know.

This yields three further adequacy conditions:

- (C7) An adequate closure schema should concern the contrastive K_{spq} state.

- (C8) An adequate closure schema should preserve epistemic modesty.
- (C9) An adequate closure schema should fit knowledge-*whether*.

C7–C9 are connected. Moore’s knowledge as to whether he has hands or stumps, and his ignorance as to whether he has hands or vat-images of hands, comprises his modest epistemic standing. There are some questions he can answer, and some he can’t. This modest standing is expressible by going explicit about the contrasts: Moore knows that he has hands rather than stumps, but Moore does not know that he has hands rather than vat-images of hands. Here is the path between dogmatism and skepticism. To fail C7–C9 is to fall into skeptical paradox.

3. CONTEXTUALIST CLOSURE

It will prove useful to turn to the standard contextualist formulations of closure, since contextualism and contrastivism are close relatives. This will help identify further adequacy conditions for closure, illustrate more useful ideas, and reveal problems for the contextualist.

The standard contextualist view is that closure holds as long as the relevant alternatives are fixed. Here is the simplest meta-linguistic implementation of this idea:

- (4) If an utterance of “*s* knows that p_1 ” is true in context c_1 with relevant alternatives R , and p_1 entails p_2 , then an utterance of “*s* knows that p_2 ” is true in any context c_2 with relevant alternatives R .

In short, the truth of knowledge ascriptions extends under entailment, as long as the relevant alternatives hold fixed.⁸

While contextualists are typically silent about the knowledge relation, they are perhaps best understood as *epistemic pluralists*.⁹ For the contextualist, “knows” is an indexical that ranges over a plurality of epistemic relations K_R . Each set of relevant alternatives R is paired with an epistemic relation K_R . So 4 can be expressed in the object-language as:

$$(5) \quad (K_{RSp_1} \ \& \ (p_1 \rightarrow p_2)) \rightarrow K_{RSp_2}$$

In words: if s knows $_R$ that p_1 , and p_1 entails p_2 , then s knows $_R$ that p_2 .

Of course, 5 fails C4–C6. The omniscience, lack-of-belief, and irrational-basing problems now arise for each K_R relation. But 3 reveals how to solve these problems. So putting 3 and 5 together, and continuing on in the object-language, yields the best contextualist closure schema:

$$(6) \quad (K_{RSp_1} \ \& \ (p_1 \rightarrow p_2)) \rightarrow E_{RSp_2}$$

That is, *position to know $_R$* is closed under entailment.¹⁰

What can be said in favor of 6 is that it satisfies C1–C7 and C9. C1 is satisfied insofar as the intuitively compelling idea of closure is preserved, in that all epistemic relations extend under entailment. C2 is satisfied since proof extends all epistemic relations. C3 is satisfied since abominable conjunctions are blocked (there is no single epistemic relation K_R for which the abominable conjunction holds. C4–C6 are satisfied by borrowing the positioning maneuver from 3. C7 is satisfied since it is trivial to translate an indexed plurality of binary relations K_{Rsp} , into the contrastive relation K_{spq} , by taking q to be the disjunction of the members of R . And C9 is satisfied as long as the content of the *whether*-question is permitted to set the relevant alternatives.¹¹ This yields Moore's knowing $_{\{\text{stumps}\}}$ that he has hands and his not knowing $_{\{\text{vat}\}}$ that he has hands, which are distinct epistemic relations, and so not forced into conflict by 6. Thus the contextualist can claim to skirt skeptical paradox.

But what can be said against 6 is that it fails C8, and that it is insufficient for many cases of knowledge expansion under deduction. Starting with C8, suppose that Moore knows $_R$ that he has hands. Then by 6, he is in position to know $_R$ that he is not a brain-in-a-vat. Thus all the K_R relations are either dogmatic or skeptical—none are modest.¹² Further, the dogmatic epistemic relations seem to involve 'cheap knowledge' without any evidential basis. Suppose, for instance, that Moore knows $_R$ that he has hands, where $R = \{\text{that Moore has stumps}\}$. To achieve this state, Moore must eliminate the alternative of

having stumps. But now consider Moore's alleged state of knowing_R that he is not a brain-in-a-vat. Not-being-a-brain-in-a-vat is no longer an *alternative* to having stumps (rather, it is a super-case of it). There is no relevant alternative to not-bring-a-brain-in-a-vat in *R*. So to achieve this alleged state, it seems that Moore need not eliminate anything. But surely epistemic achievements require an evidential basis.¹³

6 is also insufficient for many cases of knowledge expansion under deduction. As a first example, suppose I know that the murderer is Scarlet rather than Peacock or Mustard. Then I should be in position to know that the murderer is Scarlet rather than Peacock. (Equivalently in this context: suppose I know whether the murderer is Scarlet or Peacock or Mustard, then I should be in position to know whether the murderer is Scarlet or Peacock.) After all, if I can eliminate both the Peacock and Mustard alternatives, surely I can eliminate only the Peacock alternative. In general, one should be able to extend knowledge_{R1} that *p*, to knowledge_{R2} that *p*, where $R1 \supset R2$. But 6 does not apply because *R* varies. My knowledge_{Peacock, Mustard} that the murderer is Scarlet is disconnected from any knowledge_{Peacock} that the murderer is Scarlet.

As a second example of the insufficiency of 6, suppose I know that the murderer is Scarlet or Peacock, rather than Mustard, *and* that I know that the murderer is Scarlet or Green, rather than Mustard. Then I should be in position to know that the murderer is Scarlet rather than Mustard. (Equivalently in this context: suppose I know whether the murderer is (i) Scarlet or Green, or (ii) Mustard, *and* that I know whether the murderer is (i) Scarlet or Peacock, or (ii) Mustard; then I should be in position to know whether the murderer is Scarlet or Mustard.) After all, I can eliminate the Mustard possibility throughout. In general, one should be able to combine knowledge_R that *p*₁, with knowledge_R that *p*₂, to know_R that *p*₁ and *p*₂. But 6 does not apply because neither *p*₁ nor *p*₂ individually entails *p*₁ and *p*₂ — 6 has no mechanism for *multi-premise* closure.¹⁴

As a third example of the insufficiency of 6, suppose I know that the murderer is Scarlet rather than Peacock, *and* that I

know that the murderer is Scarlet rather than Mustard. Then I should be in a position to know that the murderer is Scarlet rather than Peacock or Mustard. (Equivalently in this context: suppose I know whether the murderer is Scarlet or Peacock, and that I know whether the murderer is Scarlet or Mustard; then I should be in a position to know whether the murderer is Scarlet or Peacock or Mustard.) After all, if I can eliminate both of the Peacock and Mustard disjuncts individually, surely I can eliminate their disjunction. In general, one should be able to combine knowledge_{*R*1} that *p*, with knowledge_{*R*2} that *p*, to know_{*R*1∪*R*2} that *p*. But 6 does not apply because *R* varies, and also because there are multiple premises.

These last three points yield three further adequacy conditions:

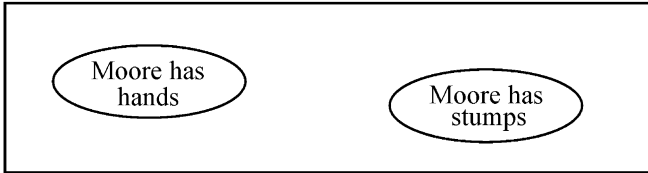
- (C10) An adequate closure schema should allow extension from knowledge_{*R*1} that *p*, to the position to know_{*R*2} that *p*, where *R*1 ⊂ *R*2.
- (C11) An adequate closure schema should allow extension from knowledge_{*R*} that *p*1 and knowledge_{*R*} that *p*2, to the position to know_{*R*} that *p*1 and *p*2.
- (C12) An adequate closure schema should allow extension from knowledge_{*R*1} that *p* and knowledge_{*R*2} that *p*, to the position to know_{*R*1∪*R*2} that *p*.

What has gone wrong is that the contextualist has added a parameter—the *R*-subscripts—but has only implemented a closure schema built for *K*_{*S**p*}.¹⁵ To fail C10–C12 is to be insufficient to cover deductive extensions of knowledge.

4. KNOWLEDGE AND ANSWERS

So how should the contrastivist formulate closure, to satisfy C1–C12? To begin, it will prove helpful to connect contrastive knowledge with *the ability to answer questions*. There are three main points of connection. First, {*p*, *q*} spans the denotation of a question. A question denotes a set of multiple alternatives—all well-formed questions are multiple-choice questions.¹⁶ The possible answers to a question form multiple

cells in a region of logical space. Questions partition the possibilities. Thus the question of whether Moore has hands or stumps can be diagrammed as:

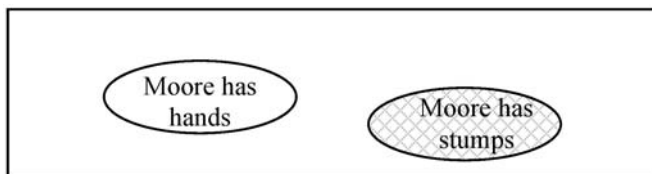


So contrastive knowledge is equivalent to question-relative knowledge: $Kspq$ is equivalent to $KspQ$, where Q is the question $?:\{p, q\}$. For instance, Moore's knowledge that he has hands rather than stumps is equivalent to his knowledge that he has hands relative to the question of whether he has hands or stumps.¹⁷

A second point of connection between contrastive knowing and answering is that the contrasts, when left implicit, are determined by the question under discussion. A context can be modeled as a set of alternatives—those alternatives that are the 'live options' under discussion.¹⁸ So when a knowledge ascription is inexplicit as to the value of q , q defaults to the disjunction of non- p options under discussion.¹⁹

A third point of connection between contrastive knowing and answering is that the elimination of the relevant alternatives constitutes the ability to answer. That is, as per A1, the evidential component of contrastive knowledge is the elimination of the alternatives. Which is what it takes to be able to answer. After all, imagine facing a multiple-choice question, and eliminating all but one option. One would be in an ideal position to answer.²⁰ Thus contrastivism implements Castañeda's idea that, "knowledge involves essentially the non-doxastic component of a power to answer a question" (1980, p. 194). All knowing is knowing the answer.

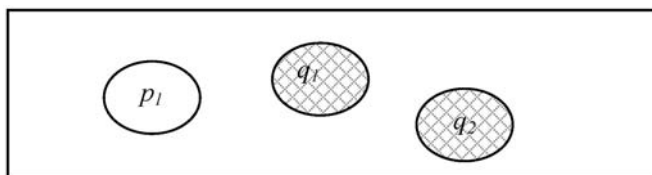
So contrastive knowledge can be represented via cells in a region of logical space, with all but one cell crossed out. So the following represents the evidential basis for Moore's knowledge that he has hands rather than stumps:



Thus the question of how the contrastivist should formulate closure may be usefully transformed into the question of how the ability to answer questions extends.

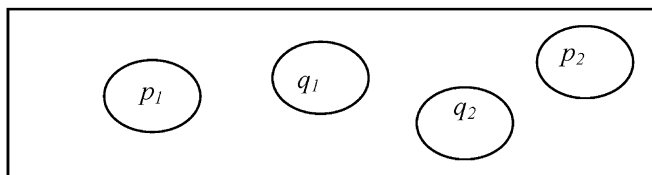
5. EXTENDING ANSWERS

So how does the ability to answer questions extend? Suppose that s is able to answer $Q_1 = \{p_1, q_1, q_2\}$, via p_1 :

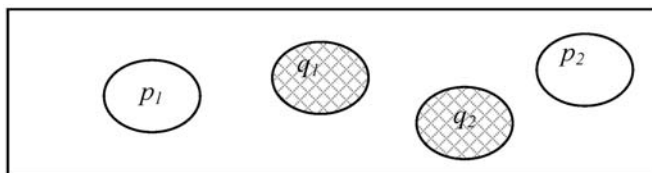


Then what other questions is s in a position to answer?

First, consider the question $Q_2 = \{p_1 \vee p_2, q_1, q_2\}$, where p_2 concerns a hitherto irrelevant region of logical space:



Since we are given that s is able to eliminate q_1 and q_2 , s is in the position represented by:



This shows that s is able to answer Q_2 via $p_1 \vee p_2$.

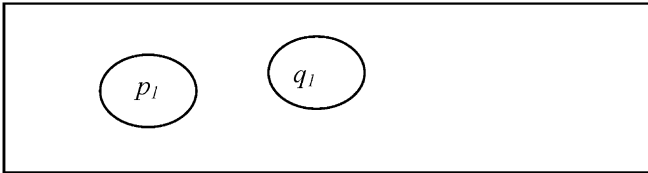
Second, consider the alleged question $Q_3 = ?:\{p_1 \vee \sim p_1, q_1, q_2\}$. Here the first ‘cell’ $\{p_1 \vee \sim p_1\}$ covers the others. So this is not a genuine question, since a genuine question requires a partition. Thus s is not able to answer Q_3 via $p_1 \vee \sim p_1$, since Q_3 is not a genuine question that can be answered.

What the extension from Q_1 to Q_2 shows is that $Kspq$ extends under p -expansion. This has a topological rationale, which is that any super-region of an uneliminated region is an uneliminated region.²¹ What the failure of extension from Q_1 to Q_3 shows is that extension under p -expansion is limited by q . If p expands to overlap q at any point, then what results is no longer a genuine question, and so no longer something capable of being answered.²² Putting this together yields a first closure schema:

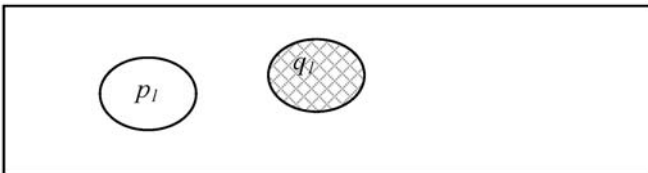
$$(\text{Expand-}p) (Ksp_1q \ \& \ (p_1 \rightarrow p_2) \ \& \ \{p_2\} \cap \{q\} = \emptyset) \rightarrow Esp_2q$$

In short, position to know is closed under q -bounded expansion at p .

Third, consider the question $Q_4 = \{p_1, q_1\}$:



Given that s is able to answer Q_1 , s can eliminate q_1 , so s is in the position represented by:



This shows that s is able to answer Q_4 via p_1 .

Fourth, consider the alleged question $Q_5 = ?:\{p_1\}$. Q_5 has only one cell. So Q_5 is not a genuine question, since a genuine

question requires multiple cells.²³ So s is not able to answer Q_5 via p_1 , since Q_5 is not a genuine question that can be answered.

What the extension from Q_1 to Q_4 shows is that $Kspq$ extends under q -contraction. There is a topological rationale for this, which is that any subregion of an eliminated region is an eliminated region.²⁴ What the failure of extension from Q_1 to Q_5 shows is that extension under q -contraction is limited by \emptyset . If q contracts to nothing, then what results is no longer a genuine question, and so no longer something capable of being answered. Putting this together yields a second closure schema:

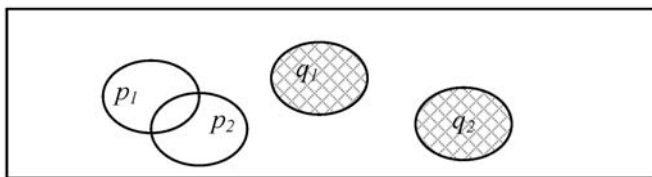
$$(\text{Contract-}q)(Kspq_1 \ \& \ (q_2 \rightarrow q_1) \ \& \ \{q_2\} \neq \emptyset) \rightarrow Espq_2$$

In short, position to know is closed under nonempty contraction at q .

6. COMBINING ANSWERS

So far, Expand- p and Contract- q describe how knowing a single answer extends. It remains to describe how knowing multiple answers combines. Here it will suffice to consider the case of knowing two answers, since all (finite) multiplicities can be accessed by repeated application of pairwise principles.

So first, suppose that s is knowingly able to answer Q_1 ($Q_1 = ?:\{p_1, q_1, q_2\}$) via p_1 , and that s is knowingly able to answer Q_6 ($Q_6 = ?:\{p_2, q_1, q_2\}$) via p_2 . From these suppositions and the factivity of knowledge ($Kspq \rightarrow p$), it follows that $\{p_1\} \cap \{p_2\} \neq \emptyset$.²⁵ And it follows that $\{p_1\} \cap \{p_2\}$ is an uneliminated region.²⁶ So s is in the position represented by:²⁷



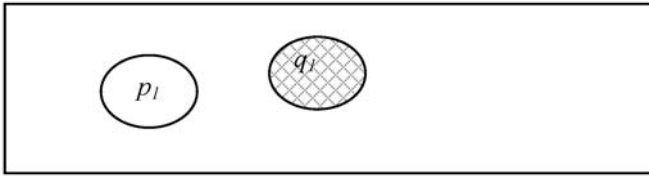
This shows that s is able to answer Q_7 ($Q_7 = ?:\{p_1 \ \& \ p_2, q_1, q_2\}$) via $p_1 \ \& \ p_2$.

What the extension from Q_1 and Q_6 to Q_7 shows is that Ksp_1q and Ksp_2q states extend under p -intersection. This yields a third closure schema:

$$(\text{Intersect-}p)(Ksp_1q \ \& \ (Ksp_2q) \rightarrow Es(p_1\&p_2)q$$

In short, combined position to know is closed under intersection at p .

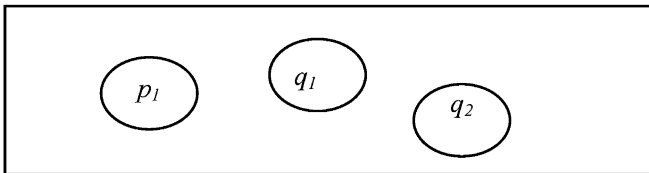
Second, suppose that s is able to answer Q_4 ($Q_4 = ?:\{p_1, q_1\}$) via p_1 , and that s is able to answer Q_8 ($Q_8 = ?:\{p_1, q_2\}$) via p_1 . Then by being able to answer Q_4 , s is in the following position:



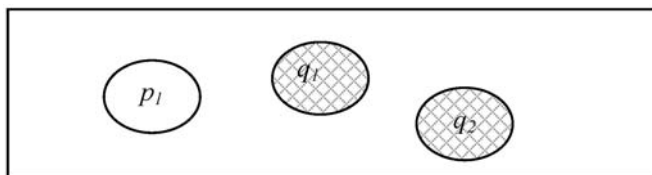
And by being able to answer Q_8 , s is in the following position:



Now consider whether s is able to answer Q_1 ($Q_1 = ?:\{p_1, q_1, q_2\}$) represented as:



Since s is able to answer Q_4 , it follows that s can eliminate q_1 . And since s is able to answer Q_8 , it follows that s can eliminate q_2 . So s is in position to eliminate both:



Which is what it takes to be able to answer Q_1 via p_1 .

What the extension from Q_4 and Q_8 to Q_1 shows is that the K_{spq_1} and K_{spq_2} states extend under q -union. This yields a fourth closure schema:

$$(\text{Union-}q)(K_{spq_1} \& K_{spq_2}) \rightarrow \text{Esp}(q_1 \vee q_2).$$

In short, combined position to know is closed under union at q .

7. CONTRASTIVE CLOSURE

So far I have presented 12 adequacy conditions on closure, and developed four closure schemata. Conjoining these closure schemata yields *contrastive closure*:

- (7) Contrastive knowledge extends by Expand- p , Contract- q , Intersect- p , and Union- q .

It remains to show that 7 satisfies C1–C12.

Starting with C1–C3, 7 preserves the closure intuitions. A plausible rendering of how knowledge extends under entailment is provided, as per C1, given the equation of contrastive knowledge with the ability to answer the question (§4), and a plausible treatment of how the ability to answer questions extends (§§5–6). Mathematical proof is allowed to extend knowledge, as per C2, since a proof can be regarded as extending the ability to answer questions. To take a trivial example, suppose I know that $2 + 2 = 4$ rather than any other natural number, and that I prove that $2 + 2 = 4$ entails that $2 + 2 = 3 + 1$. Then by Expand- p I am in position to know that $2 + 2 = 3 + 1$ rather than any other natural number.²⁸ Abominable conjunctions are blocked, as per C3, since there is no single value of q that can vindicate: “ s does not know that she is not a brain-in-a-vat [rather than q],

but *s* does not that she has hands [rather than *q*].” For the second conjunct requires that *q* be eliminable, and the first conjunct requires that *q* contrast with not being a brain-in-a-vat. So *q* would need to be an eliminable case of being a brain-in-a-vat. Thus there is no such *q*.

Turning to C4–C6, 7 avoids the counterexamples. By conjoining principles with *Espq* consequents, 7 draws on the maneuver that avoids the counterexamples (§2). Lack of omniscience, belief-failure, and basing-failure cannot imperil any component of 7.

Moving to C7–C9, 7 skirts skeptical paradox. 7 fits contrastivism directly, as per C7. Epistemic modesty is preserved, as per C8. By restricting Expand-*p* so that what is entailed cannot overlap the contrasts, cases of ‘cheap knowledge’ with nothing to eliminate are blocked. Thus one cannot use one’s knowledge that one has hands rather than stumps, to come to know that one is not a brain-in-a-vat rather than a creature with stumps.²⁹ And since there is no rule of Replace-*q*, one cannot use one’s knowledge that one has hands rather than stumps ($K.spq_1$), to come to know that one has hands rather than vat-images of hands ($K.spq_2$).³⁰ So Moore knows that he has hands rather than stumps, and does not know that he has hands rather than vat-images of hands. And knowledge-*whether* is accommodated, as per C9. Moore knows whether he has hands or stumps, and does not know whether he has hands or vat-images of hands.³¹

Finishing with C10–C12, 7 licenses the needed inferences. C10 requires the extension from knowledge_{*R1*} that *p*, to the position to know_{*R2*} that *p*, where $R1 \supset R2$. This is what Contract-*q* licenses. So if I know that the murderer is Scarlet rather than Peacock or Mustard, then, by Contract-*q*, I am in position to know that the murderer is Scarlet rather than Peacock. C11 requires the extension from knowledge_{*R*} that *p*₁ and knowledge_{*R*} that *p*₂, to the position to know_{*R*} that *p*₁ & *p*₂. This is what Intersect-*p* licenses. So if I know that the murderer is Scarlet or Peacock, rather than Mustard, and I know that the murderer is Scarlet or Green, rather than Mustard, then, by Intersect-*p*, I am in position to know that the

murderer is Scarlet rather than Mustard. C12 requires the extension from knowledge_{RI} that p and knowledge_{R2} that p , to the position to know_{RI∪R2} that p . This is what Union- q licenses. So if I know that the murderer is Scarlet rather than Peacock, *and* I know that the murderer is Scarlet rather than Mustard, then I am in a position to know that the murderer is Scarlet rather than Peacock or Mustard.

No doubt there are further counterexamples to be fielded, and further inferences to be supported. Still I would conclude that contrastivism delivers the best account of closure to date.³²

NOTES

¹ The contrastive theory emerges in the following passage from Dretske: “To know that x is A is to know that x is A within a framework of relevant alternatives, B , C , and D . This set of contrasts ... serve to define what it is that is known...” (1970, p. 1022) The theory has since been defended by Johnsen (2001), Morton and Karjalainen (2003), Sinnott-Armstrong (2004), and Schaffer (2004a, 2005b). In this vein, Johnsen maintains: “what is known is always a contrastive proposition to the effect that P-rather-than-any-other-member-of-category-C is true...” (2001, p. 401).

² Dretske (1970), Nozick (1981), and Heller (1999a) have argued against any form of closure. I will not be addressing their arguments.

³ Hintikka (1962) treats epistemic logic via the S4 axioms:

$$D: Ks(p_1 \rightarrow p_2) \rightarrow (Ksp_1 \rightarrow Ksp_2)$$

$$T: Ksp \rightarrow p$$

$$B: Ksp \rightarrow KKsp$$

2 follows from D. (*Proof:* suppose Ksp_1 and $Ks(p_1 \rightarrow p_2)$. From $Ks(p_1 \rightarrow p_2)$ and D, we get $Ksp_1 \rightarrow Ksp_2$. From this and Ksp_1 we get Ksp_2 . So $(Ksp_1 \ \& \ Ks(p_1 \rightarrow p_2)) \rightarrow Ksp_2$.) Thus 2 is a consequence of modeling epistemic logic as a normal modal system, for which the distribution axioms hold.

⁴ *Complication:* C3 is not perfectly satisfied with 2 (and likewise for all subsequent schemata), since one can fail to know that one is not a brain-in-a-vat and still know that one has hands, if one does not see the entailment relation here. But this is the right result. Really the conjunction is only abominable for the subject who sees the connection between handedness and not being envatted. The abominable conjunction can hold true of abominable reasoners.

⁵ The idea of ‘position to know’ is invoked in Cohen (1999, p. 84). The understanding of position-to-know as only needing a belief-based-on-competent-deduction-while-retaining-knowledge-of-the-premises in order to know, is drawn from Williamson (2000) and Hawthorne (2004). This represents the state-of-the-art for binary closure.

⁶ Dretske rejects closure for essentially this reason. According to Dretske, one should be able to know that the animal is a zebra, without being in position to know that the animal is not a cleverly painted mule. Of course, most binary theorists will have already accustomed themselves to accepting that one can know that one is not a brain-in-a-vat, and that one can know that the animal is not a cleverly painted mule. But I think this should be reckoned a counterintuitive consequence.

⁷ For instance, Lewis writes: “Holmes knows whether ... if and only if he knows the true one of the alternatives presented by the ‘whether’-clause, whichever one that is.” (1998, p. 45) This is an instance of the standard binary view of knowledge-*wh* constructions. Here Higginbotham provides the rule: “ $\text{know}(x, \hat{\pi}) \leftrightarrow (\exists p) (\text{know}(x, p) \ \& \ p \text{ answers } \pi)$ ” (1982, p. 194). See Schaffer (*forthcoming*) for further discussion.

⁸ The idea of closure for fixed alternatives is due to Stine (1976), and has been upheld by leading contextualists since, including Cohen (1988), DeRose (1995), and Lewis (1996).

⁹ Heller is explicit here: “What contextualism tell us is that ‘knowledge’ refers to different properties in different contexts.” (1999b, p. 115) And: “Every selection of worlds defines a specific epistemic property. It is these various properties that are the possible referents for the term ‘knowledge’.” (1999b, p. 117) See Schaffer 2004b (§2) for further discussion.

¹⁰ The understanding of contextualist closure via 6 is contentious in two main respects. First, not all contextualists will want to index to relevant alternatives. Some contextualists, such as Cohen 1999, will prefer to index to epistemic standards (which concern the degree of evidence sufficient to satisfy ‘justified’). So this sort of contextualist might replace 6 with:

$$(6s) \ (K_s p_1 \ \& \ (p_1 \rightarrow p_2)) \rightarrow K_s p_2$$

Where *S* denotes an epistemic standard. 6s will function roughly equivalently to 6 from the perspective of the adequacy conditions of the main text, save that it will render the satisfaction of C9 more mysterious (see footnote 11). In the main text I will continue to presume that the contextualist is a relevant alternatives theorist. For arguments in favor of the relevant alternatives style of contextualism, see Schaffer (2005a).

The second respect in which the understanding of contextualism in 6 is contentious is that some contextualists (Cohen, 1999, p. 61) will also count ternary theories (including contrastivism) as forms of ‘contextualism’. So this sort of theorist might count the following as a variant form of ‘contextualism’:

$$(6t) (K_{sp_1}R \ \& \ (p_1 \rightarrow p_2)) \rightarrow K_{sp_2}R$$

By treating the contrast proposition q as the disjunction of the relevant alternatives in R , one can see that 6t is equivalent to the following contrastive schema (which is a simplified version of Expand- p : §5):

$$(6c) (K_{sp_1}q \ \& \ (p_1 \rightarrow p_2)) \rightarrow K_{sp_2}q$$

Or, replacing relevant alternatives in 6t with standards:

$$(6ts) (K_{sp_1}S \ \& \ (p_1 \rightarrow p_2)) \rightarrow K_{sp_2}S$$

The issue of what counts as ‘contextualism’ here is purely terminological. I am interested in distinguishing between (i) the pluralist view that there are many binary K relations (in 6 and 6s), and (ii) the ternarist view that there is only one ternary K relation (in 6t, 6c, and 6ts). I am using ‘contextualism’ to cover (i) and reserving ‘contrastivism’ to cover the case of (ii) where the third argument place is a contrast slot. The philosopher who prefers a more general construal of ‘contextualism’ is welcome to regard all these theories as species of ‘contextualism’ (she might then distinguish ‘pluralist contextualism’ from ‘contrastivist contextualism’). I only ask that she mark the distinction here, in whatever terminology she prefers.

¹¹ Though it is not obvious that the contextualist is entitled to this. For instance, if the contextualist follows anything like the relevance rules of Lewis (1996), then what her rules deliver will not match what the *whether*-clause queries (Schaffer, 2004a). While if the contextualist prefers to index to epistemic standards (see footnote 10), then she will need to forge a connection between the alternatives of the *whether*-question and the standards. It is not at all obvious how this would go. I will waive this concern in the main text.

¹² The best the contextualist can do is to rig the rules of relevance to create the *appearance* of modesty, by having all assertions of modest knowledge set the context to a dogmatic epistemic relation K_{R1} , and all assertions of modest ignorance set the context to a skeptical epistemic relation K_{R2} .

¹³ Here I am drawing on Heller (1999a) and Kvanvig (*manuscript*). For Heller, the resulting ‘cheap knowledge’ constitutes a reason to discard closure entirely. For Kvanvig, it constitutes a decisive objection to contextualism and contrastivism alike. Kvanvig gives the case where s knows that Fido is a dog rather than a cat, claims that closure should allow s to know that Fido is an animal rather than a cat, and charges that the contrastivist can make no sense of this. My reply is that contrastive closure (as formulated below: §7) in fact does *not* allow s to know that Fido is an animal rather than a cat. This is the right result,

since such a state is indeed nonsensical: being an animal does not contrast with being a cat.

¹⁴ Here I am adding the assumption that *multi-premise closure* should hold. I take the example in the main text to illustrate the plausibility of this assumption. For further defense see Hawthorne (2004, pp. 46–50).

¹⁵ The problem is that the *R*-subscripts are blocking what would otherwise be valid inferences. That is, the inferences in C10–C12 are valid under the binary closure schema 3. With 3, C10 becomes the valid inference from K_{sp} to E_{sp} ; C11 becomes the valid multi-premise inference from $(K_{sp} \& K_{sq})$ to $E_{s(p\&q)}$; and C12 becomes the valid inference from $(K_{sp} \& K_{sp})$ to E_{sp} . Thus 6 cannot do the work that 3 does. The contextualist needs to regain these lost inferences.

¹⁶ The association of questions with multiple-choice slates is known as *Hamblin's dictum* (Hamblin, 1958), and is implemented in Belnap and Steel's (1976) erotetic logic, and maintained in the partition semantics of Groenendijk and Stokhof (1997).

¹⁷ If the question Q has more than two answers, K_{spQ} contains more information than K_{spq} , since Q partitions the contrasts, while q lumps them into one big disjunction. But this additional information plays no epistemic role. All the alternatives must be eliminated, however they are partitioned.

¹⁸ The strategy of modeling contexts in terms of sets of alternatives is due to Stalnaker (1999a, b). For general linguistic evidence that the conversational score includes an entry for *question under discussion*, see Levinson (1983) and Ginzburg (1996).

¹⁹ When a knowledge ascription is explicit as to the value of q , via such mechanisms as explicit “rather than”-clauses, interrogative complements (including knowledge-whether constructions), or contrastive stress, q is to be set to its explicit value. Note that most knowledge ascriptions involve interrogative complements. Here are some common examples: “I know what time it is,” “You know who wrote the paper,” and “She knows where to go.” Note also that most spoken knowledge ascriptions with declarative complements feature contrastive stress. Thus compare: “I know that *John* kissed Mary,” “I know that John *kissed* Mary,” “I know that John kissed *Mary*.” The unstressed binary form, as in “I know that John kissed Mary,” thus seems relatively rare. Most ordinary knowledge ascriptions are explicitly contrastive.

²⁰ As the referee for *Philosophical Studies* noted, one might also want some positive grounds for the remaining option. If so, one should take E_{spq} to require both the elimination of q , and some positive grounds for p (see Schaffer, 2005b, §4). This would require more complicated diagrams, to encode the distinction between merely uneliminated versus positively supported cells. I would reach the same conclusions, but only given two further assumptions: (i) some positive support for a region

entails some positive support for its super-regions, and (ii) some positive support for two compatible regions entails some positive support for their intersection. In the main text, I will work with the simpler treatment.

²¹ *Proof:* Let p_1 be an uneliminated region and let p_2 be a super-region of p_1 . Since p_1 is uneliminated, there is at least one possibility w in p_1 which is uneliminated. Since p_2 is a super-region of p_1 , and w is in p_1 , it follows that w is in p_2 . So there is at least one possibility in p_2 which is uneliminated, namely w . So p_2 is an uneliminated region.

²² What if p expands only to partially overlap q ? Intuitively, if I know that the murderer is Scarlet rather than Peacock or Mustard, then I am in position to know that the murderer is (Scarlet or Peacock), rather than Mustard. This inference is allowed via two steps. First (as will emerge below), one can contract q . So if one knows that the murderer is Scarlet rather than Peacock or Mustard, one can know that the murderer is Scarlet rather than Mustard. One can then expand p , to know that the murderer is (Scarlet or Peacock), rather than Mustard. Note that this will only work for partial overlap (as it should). In cases where p expands to swallow q completely, no contraction of q short of \emptyset will allow for subsequent p expansion without overlap. And (as per below) it is a constraint on q -contraction that q remain nonempty.

²³ Q_5 is rather what is called a ‘merely rhetorical question’. In only allowing one option, it does not call for an answer. Rather its discourse function is that of a (veiled) assertion.

²⁴ *Proof:* Let q_1 be an eliminated region and let q_2 be a subregion of q_1 . Since q_1 is eliminated, every possibility w in q_1 is eliminated. Since q_2 is a subregion of q_1 , every possibility in q_2 is a possibility in q_1 , and is thus eliminated. So q_2 is an eliminated region.

²⁵ *Proof:* Given $Ksp_1(q_1 \vee q_2)$ and factivity, it follows that α (the actual world) $\in \{p_1\}$. Likewise, given $Ksp_2(q_1 \vee q_2)$ and factivity, it follows that $\alpha \in \{p_1\}$. So $\alpha \in \{p_1\} \cap \{p_2\}$, so $\{p_1\} \cap \{p_2\} \neq \emptyset$.

²⁶ *Proof:* From the previous proof, we have. $\alpha \in \{p_1\} \cap \{p_2\}$ But α is uneliminated by definition (Lewis, 1996), so there is an uneliminated possibility in $\{p_1\} \cap \{p_2\}$, so $\{p_1\} \cap \{p_2\}$ is an uneliminated region.

²⁷ I have diagrammed the case where neither p_1 nor p_2 is a part of the other. But the same point holds if either is a part of the other. Actually the result will be trivial in that case, since the Intersect- p rule (below) will output knowledge of the part, which will be the same bit of knowledge as was originally supposed. (That is, if p_1 is a part of p_2 , then Intersect- p will output Ksp_1q , which is just the first supposition.)

²⁸ Though here the possible world treatment of propositions goes awry, since mathematical propositions hold at all worlds or none. This is a problem for everyone. I take no position here on how to solve it here.

²⁹ Likewise for Kvanvig's example (as per footnote 13), one cannot use knowledge that Fido is a dog rather than a cat, to come to know that Fido is an animal rather than a cat. After all, "Is Fido an animal or a cat?" does not constitute a genuine question. And the state of knowing whether Fido is an animal or a cat does not constitute a genuine state.

³⁰ *Complication:* q -expansion is permitted in the combined case, via Union- q . In order to use Union- q to replace one's modest knowledge that one has hands rather than stumps, with immodest knowledge that one has hands rather than vat-images of hands, one would need a second piece of knowledge. This second piece of knowledge would need to be knowledge that one has hands rather than q_2 , where $q_1 \in q_2$ would be the brain-in-a-vat possibility (or a superset thereof, with Contract- q then being applied to pare down to the brain-in-a-vat possibility). So q_2 would need to contain the brain-in-a-vat hypothesis. But that would make it ineliminable. Thus no such second piece of knowledge exists.

³¹ Similar comments apply to Dretske's example (as per footnote 6). One can know that the animal is a zebra rather than a normal mule, *and* not know that the animal is a zebra rather than a cleverly painted mule. Further, one can know whether the animal is a zebra or a normal mule, *and* not know whether the animal is a zebra or a painted mule.

³² Thanks to Martijn Blaauw, Stewart Cohen, Bjørn Jespersen, Jonathan Kvanvig, Ram Neta, the referee for *Philosophical Studies*, and the audience for the Contextualism Conference at the Free University of Amsterdam.

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