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Deterministic Chance?

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ABSTRACT

Can there be deterministic chance? That is, can there be objective chance values other than 0 or 1, in a deterministic world? I will argue that the answer is *no*. In a deterministic world, the only function that can play the role of chance is one that outputs just 0s and 1s. The role of chance involves connections from chance to credence, possibility, time, intrinsicness, lawhood, and causation. These connections do not allow for deterministic chance.

Can there be *deterministic chance*? That is, can there be objective chance values other than 0 or 1, in a deterministic world? I will argue that the answer is *no*. In a deterministic world, the only function that can play the role of chance is one that outputs just 0s and 1s.

1 Overview

It was once widely assumed that determinism and chance were incompatible. That is, it was once widely assumed that deterministic worlds could only have extremal (0 or 1) objective chances. Thus Popper declares:

> Today I can see why so many determinists, ... believe in a subjectivist interpretation of probability: it is, in a way, the *only reasonable possibility* which they can accept; for objective physical probabilities are incompatible with determinism; ... ([1982], p. 105)

And Lewis insists:

> If the chance [of heads up] is zero or one, ... then it cannot also be 50%. To the question of how chance can be reconciled with determinism, or to

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> the question of how disparate chances can be reconciled with one another, my answer is: *it can't be done*. ([1986], p. 118)

Neither Popper nor Lewis offers much more by way of argument here. Presumably, they did not feel the need.

But now it is widely allowed that determinism and chance are compatible. For instance, Clark writes: 'It seems to me that the issue of determinism versus indeterminism really ought to be (is) irrelevant to an interpretation of probability theory' ([2001], p. 275). Hoefer says: 'whatever objective chances are, they are certainly compatible with the reign of determinism at the level of physical law' ([unpublished]). And Loewer offers an account of 'the conditions that we would like an adequate account of chance to satisfy,' which includes: 'Chances different from 0 and 1 are compatible with determinism' ([2001], p. 613). So the pendulum has swung.[1]

In what follows, I will defend incompatibilism. Here is the plan. In Sections 2 and 3, I will set the stage by discussing four compatibilist arguments in the literature, leading to four conceptions of what deterministic chance might be. In Sections 4 to 6, I will connect determinism and chance. Here, connections will be traced from chance to credence, possibility, futurity, intrinsicness, lawhood, and causation, which will then be used to argue that none of the four conceptions of deterministic chance can fit the chance role. Finally, in Section 7, I will clarify the distinction between objective chance and epistemic chance—for I suspect that the compatibilist may have conflated these notions.

2 Four Arguments for Deterministic Chance

Why believe in deterministic chance? It will prove useful to begin by reviewing the arguments for deterministic chance. This will consolidate some of the literature, explain what motivates the compatibilist, and set the agenda for the incompatibilist.

So, the first argument for compatibilism is *the no-connection argument*. The no-connection argument holds that the intended notions of determinism and of chance have no bearing on each other. On the intended notion of determinism, a world is deterministic iff its total occurrent history is fixed by its occurrent state at any time plus its laws:

[1] Or, so the pendulum has *swung back*. For deterministic chance is allowed on the old *frequency* conception of chance—determinism allows that the actual proportion of heads landings to coin flippings be 0.5.

> *Determinism*: A world w is deterministic iff: for all times t in w, the total occurrent history of w supervenes on the occurrent state of w at t together with the laws of w.[2]

What is essential here is that determinism, so characterized, is just a supervenience thesis.

On the intended notion of chance, chance is a function from propositions, worlds, and times onto the closed unit interval, in accord with the probability axioms. Formally speaking, this renders chance as:

> *Chance-formal*: Chance is a probability function: $ch\langle p, w, t\rangle \rightarrow [0, 1]$.[3]

Here '$ch\langle p, w, t\rangle$' may be read as 'the chance that p obtains in world w, assessed at time t'. In what follows, I will mainly be concerned with the chances of individual, momentary event occurrences. I will use the following notational conventions. Event subscripts on p denote propositions about the occurrence of the subscripted (individual, momentary) event. So if *heads* is the event of the coin landing heads at midnight, p_{heads} is the proposition that the coin lands heads up at midnight. Event subscripts on t denote the moment at which the (individual, momentary) event is targeted to occur (when it would occur, if

2 For related characterizations of determinism, see (Earman [1986]; Hoefer [2004]). According to Earman, 'The world $w \in W$ (the set of physically possible worlds) is *Laplacian deterministic* just in case for any $w' \in W$, if w and w' agree at any time, then they agree for all times' ([1986], p. 13). This characterization can be weakened to yield a family of weaker notions of determinism, including:

> *Forward determinism*: A world w is forward-deterministic iff: for all times t in w, the occurrent post-t future of w supervenes on the occurrent state of w at t together with the laws of w.

> *Regional determinism*: A region r of world w is regionally deterministic iff: for all times t in r, the total occurrent history of r supervenes on the occurrent state of r at t together with the laws of w.

> *Aspectual determinism*: A world w is deterministic with respect to occurrent property p iff: for all times t in w, the total p-history of w supervenes on the p-state of w at t together with the laws of w.

Note that, as defined in the main text, a world can fail to be deterministic in several ways: (i) by having laws that allow multiple outcomes, (ii) by having laws with a chance distribution over multiple outcomes, and (iii) by not having laws at all governing certain transitions. There can also fail to be a fact about whether a world is deterministic, if there can fail to be a fact about what laws it has. The relevant question is whether a world that is in fact deterministic can also feature nonextremal objective chances.

3 Treating the input to the chance function as a $\langle p, w, t\rangle$ triple embodies the substantive assumption that the chance function needs no further inputs (such as a reference class). This assumption is (for the most part) common ground in the deterministic chance debate. It will be defended indirectly in what follows, insofar as the role of chance (Section 4) will prove explicable in ways that require no further inputs. In any case, whether determinism is compatible with a 'chance' function that is relativized to reference classes or other further inputs should be considered a separate question not addressed in the main text.

it does occur). So if *flip* is the event of the coin being flipped at one second before midnight, t_{flip} is one second before midnight, and t_{heads} is midnight. (Of course *heads* need not occur at w for the triple $<p_{heads}, w, t_{flip}>$ to have a well-defined chance value. The coin may well land tails, but still have a 0.5 chance of landing heads at the time it is flipped.)

Now, *Chance-formal* is not a full definition, for there are many functions that satisfy this purely formal description. Substantive constraints are needed, concerning which formally eligible functions can serve as the chance function (Section 4). But what is essential here is that chance, so formalized, is just a probability function.

What emerges is that *Determinism* and *Chance-formal* have no connection. *Determinism* is a thesis of the supervenience of history on states and laws, while *Chance-formal* describes a probability function from propositions, worlds, and times onto the closed unit interval. Why should a supervenience thesis constrain a probability function at all, much less in so stringent a way as to force it to output only 0s and 1s?

This no-connection argument places the burden of proof on the incompatibilist, who must display the connection here. Much of what follows may be read as an extended answer to this argument. (This will be the thrust of Sections 4 to 6, where I introduce substantive constraints on being the chance function that go beyond *Chance-formal*, and connect these to *Determinism*.)

The second argument for deterministic chance is *the paradigm case argument*. The paradigm case argument holds that events like coin flip outcomes are paradigms of (nonextremal, objective) chance, and that such events are found at deterministic worlds.[4] So, focusing on coin flips, the second argument might run:

1. There are coin flips in deterministic worlds;
2. Coin flips involve a 0.5 objective chance that a heads landing will occur, at the world in question, at the time of flipping: $ch<p_{heads}, w, t_{flip}> = 0.5$.
3. So: there are objective chance values other than 0 or 1, in deterministic worlds.

[4] In this vein, Eagle notes:

> [P]robability analyses need to be responsive to commonsense intuition. To reject probability for coins, dice or roulette is to clash drastically enough with the pre-theoretical concept that we might think some other concept is really being elucidated ... ([2004], p. 386; see also Levi [1990], p. 142; Hoefer [unpublished])

Of course, the kind of coin flips that Eagle and others are concerned with are *unbiased* flips (or at least ones that are not completely rigged, such as involving a two-headed coin, or a 'flip' in which the coin is directly placed heads up). It is not clear if this notion of bias can be explicated without making reference to chance. But in the main text I will give the defender of the paradigm case argument the benefit of the doubt.

Here, 1 should be uncontroversial. 2 might seem supported by paradigm case considerations. And 3 follows, since 1 and 2 entail that there are deterministic worlds with 0.5 objective chance values.[5]

The incompatibilist should deny 2, and should distinguish between *objective* chance, and merely *epistemic* chance. She may then allow that coin flips are paradigm cases of chance in some form, but maintain that the chance involved is epistemic. Thus she may agree with the conclusion Pope drew from Newtonian mechanics:

> All nature is but art, unknown to thee;
> All chance, direction, which thou canst not see;
> (*Essay on Man*: I.289)

So I would conclude that paradigm case intuitions are not at stake—what is at stake is merely the *interpretation* of these intuitions as objective or epistemic.

Of course, this is not to deny that objective information (such as frequency data, and physical information concerning asymmetries in the coin) can impact the chance of landing heads. So it is with epistemic chances generally. Information impacts what we know. Here the incompatibilist must uphold a distinction between objective chance, and objectively informed but (still) epistemic chance. (This will be the topic of Section 7.)

The third argument for deterministic chance is *the nonreductionist argument*. This argument maintains that, even if some form of determinism held in the micro-realm, and even granting that this entails only extremal micro-chances, this would have no bearing on the chances applicable to macro-events, since the macro-realm has some sort of independent reality (Hoefer [unpublished]). So this third argument might run:

4. There are macro-events in deterministic worlds;
5. The macro-realm has an independent reality with independent laws and chances;
6. So: there may be macro-chances in deterministic worlds.

Of course, such a nonreductionist stance is controversial. And it is difficult to explicate the notion of 'an independent reality' in any precise way. But here it is worth supposing that macro-events exist independently in some sense, if only to see what follows.

The incompatibilist should still deny 5, however precisely explicated. That is, even granting some form of 'independent reality' to the macro-realm, the incompatibilist should still deny the independence of macro-chances.

[5] A related version of the paradigm case argument, mentioned to me by Michael Strevens and by one of the referees, looks not to coin flips but to probabilities postulated in genetics and other special sciences. The reply is the same to both versions of the argument.

For even the most robust nonreductionist should still believe in *nomological supervenience* between the micro and the macro, to explain their correlation.[6] But nomological supervenience is sufficient to correlate chance values. For if the micro-chance that micro-state $s1$ occurs at $<w, t>$ is 1, and if $s1$ subvenes macro-state $s2$ by the laws of w, then the macro-chance that $s2$ occurs at $<w, t>$ must also be 1—the occurrence of the macro-state is fixed by the fixed occurrence of its micro-basis.[7] So I would conclude that the status of reductionism is not at stake. Even the most robust nonreductionist should still accept the *nomic correlation* of micro and macro chances, just as they accept the nomic correlation of micro and macro existences.

The fourth argument for deterministic chance is *the classical statistical mechanics* (CSM) *explanation argument*. CSM looks to posit a nonextremal chance measure over events in a deterministic world.[8] The argument then maintains that this measure must be taken as an *objective* chance measure, to make sense of its role in explanation. Thus Loewer writes:

> Consider, for example, the statistical mechanical explanation of why an ice cube placed in warm water melts in a short amount of time. The explanation roughly proceeds by citing that the initial macro-conditions of the cup of water + ice ... is one in which on the micro-canonical probability distribution it is overwhelmingly likely that in a short amount of time the ... ice cube melted. If the probability appealed to in the explanation is merely a subjective degree of belief then how can it account for the melting of the ice cube? What could your ignorance of the state of the gas have to do with an explanation of its behavior? ([2001], p. 611; see also Albert [2000], p. 64)

6 For instance, Chalmers—who is robustly nonreductionistic about consciousness—maintains that the phenomenal supervenes on the functional by law ([1996], p. 124). In general, I take it as a condition of empirical adequacy on nonreductionism for a given macro-domain d, that the nomological supervenience of d on the micro-domain is upheld.

7 In general, if $s1.1$-$s1.n$ exhaustively nomologically subvene $s2$ by the laws of w, then, for all t, $ch<(s1.1 \text{ occurs } \vee s1.2 \text{ occurs } \vee \ldots \vee s1.n \text{ occurs}), w, t> = ch<s2 \text{ occurs}, w, t>$. The chance of the supervening event occurring is nomologically correlated with the chance of one of its bases occurring. I take this to be the natural generalization of the claim that nomological supervenience is sufficient to correlate existences in nonchancy worlds. According to the existence correlation claim being generalized on, if $s1.1$-$s1.n$ exhaustively nomologically subvene $s2$ by the laws of w, then it is true at w that ($s1.1$ occurs $\vee\, s1.2$ occurs $\vee \ldots \vee\, s1.n$ occurs) iff $s2$ occurs.

8 *Caveat*: CSM posits a Newtonian world, and Newtonian worlds are not really deterministic (Earman [1986], ch. 3). Though presumably the ways in which Newtonian worlds fail to be deterministic are not crucial to whatever explanatory powers CSM might possess. For instance, presumably the explanatory powers of CSM do not require the possibility of 'space invaders' appearing from infinity on trajectories asymptotic to a given time t (as per the example of Earman [1986], p. 34). So, presumably, we could imagine a modified CSM that invoked truly deterministic micro-posteriors, and the same argument would arise for CSM. Or at least, I will give the defender of deterministic chance the benefit of the doubt here.

So this fourth argument might run:

7. CSM postulates nondegenerate chances in a deterministic world;
8. Such chances play a role in explanations;
9. So: such chances must be objective chances.

Here, 7 should be uncontroversial (*modulo* issues about Newtonian mechanics). 8 is supported by scientific practice. And 9 might seem to follow, as per Loewer's point that mere subjective or epistemic chances cannot explain melting of ice cubes.

The incompatibilist should deny that 9 follows. She should distinguish between *probabilistic explanation*, in which objective chances play an explanatory role, and *probability of explanation*, which is merely an ignorance measure over various nonchancy explanatory paths. She may then allow that CSM chances play a role in explanation (as per 8), but only via probability of explanation. Presumably there will exist an explanation of whatever happens to the ice when it hits the water. The compatibilist sees an objective chance explaining the melting. The incompatibilist only sees an epistemic chance with an objectively informed measure, with a near-1 probability assigned to melting, and a near-0 probability assigned to the ice cube's freezing even further. Here too, the incompatibilist must uphold a distinction between objective chance, and objectively informed but (still) epistemic chance (Section 7).

The incompatibilist view of the role of CSM chances in explanation strikes me as plausible. If one understands CSM chances as arising from a probability distribution over a coarse-graining of initial conditions (see below), then it seems as if such chances are just expressing our ignorance over the actual, precise initial conditions. For instance, if the gas molecules in a (sealed, elastic-walled) box are in fact at precise distribution d, and if the Newtonian laws entail that d evolves into d', then this fixes what will be and identifies the real causal processes involved. The CSM chance is only telling us how to guess at what will be, and how to guess at the sort of causal process involved, absent the precise knowledge. It is merely a statistical average.[9] In any case, I would conclude that the vindication of scientific practice is not at stake here—what is at stake is merely the *interpretation* of the practice as involving probabilistic explanations or probabilities of explanations.[10]

[9] Albert acknowledges that the ignorance interpretation of CSM represents 'more or less what the [statistical] postulate amounts to ..., in the imagination of most physicists' ([2000], p. 62).

[10] The issue of how CSM chances relate to explanations connects to further issues such as the role of CSM chances in laws and counterfactuals. Thus, suppose CSM, and consider the counterfactual:

> *Melt*: If an ice cube were placed in a cup of warm water, then the chance that the ice cube melts would be very high, but less than 1.

This concludes my review of the case for deterministic chance. What emerges is that the incompatibilist has work to do. She must connect determinism to chance (Sections 4–6), and uphold a principled distinction between objective chance and objectively informed epistemic chance (Section 7).

3 Four Conceptions of Deterministic Chance

One more piece of stage setting is needed. For the four arguments for deterministic chance (Section 2) turn out to yield four distinct conceptions of what deterministic chance might be, in ways that prove to complicate the incompatibilist's task of connecting determinism to chance.

To begin with, one might distinguish *micro-* from *macro*-chances, by whether the proposition p concerns a micro- or macro-event:

Micro-chance: $\text{ch}<p_e, w, t>$ is a micro-chance iff e is a micro-event.

Macro-chance: $\text{ch}<p_e, w, t>$ is a macro-chance iff e is a macro-event.[11]

So, for instance, if *heads* is the macro-event of a given coin landing heads at midnight, then $\text{ch}<p_{heads}, w, t>$ is a macro-chance. While if *electron* is the micro-event of a given electron having a certain velocity at midnight, then $\text{ch}<p_{electron}, w, t>$ is a micro-chance.

Further, one might distinguish *initial* from *posterior* chances, by whether the proposition p concerns an event targeted for the first moment of the universe (t_0), or at some posterior time:

Initial chance: $\text{ch}<p_e, w, t>$ is an initial chance iff $t_e = t_0$.

Posterior chance: $\text{ch}<p_e, w, t>$ is a posterior chance iff $t_e > t_0$.[12]

The advocate of objective CSM chances will, presumably, say that *Melt* is true. While the advocate of the ignorance interpretation should say that *Melt* is false. On the ignorance view, the antecedent of *Melt* is merely imprecise. On one sort of precisification, corresponding to an anti-entropic set-up, the chance that the ice cube melts would be 0. While on the other sort of precisification, corresponding to an entropic set-up, the chance that the ice cube melts would be 1. On no precisification is the chance that the ice cube melts very high, but less than 1. Just clarify the antecedent, and the alleged 'chance' is what melts. To my mind this is the proper view of *Melt*. But in any case, what should be emphasized here is that what is at stake is not the science itself, but only its philosophical interpretation.

11 Recall (Section 2) that p_e is the proposition that the individual, momentary event e occurs. The micro/macro distinction only applies to those propositions that are about the occurrence of individual, momentary events—the distinction is not intended to be exhaustive. For instance, if e is the fusion of a micro- and a macro-event, then $\text{ch}<p_e, w, t>$ is neither a micro- nor a macro-chance.

12 Recall (Section 2) that t_e is the time that e is targeted to occur. The initial/posterior distinction only applies to those propositions that are about the occurrence of individual, momentary events, and to those worlds that have initial conditions (though one could naturally extend the distinction so that, in worlds without initial conditions, all events count as posterior)—the distinction is not intended to be exhaustive.

So, for instance, if *BigBang* is the initial event of the Big Bang having a certain quantum of energy, then ch$<p_{BigBang}, w, t>$ is an initial chance. While ch$<p_{heads}, w, t>$ is a posterior chance.

With these distinctions in hand, one might distinguish four conceptions of what deterministic chance might be:

> Deterministic *micro-posterior* chance: ch$<p_e, w, t>$ is a deterministic micro-posterior chance iff: (i) e is a micro-event, (ii) w is deterministic, (iii) $t_e > t_0$, and (iv) $0 < ch<p_e, w, t> < 1$.
>
> Deterministic *macro-posterior* chance: ch$<p_e, w, t>$ is a deterministic macro-posterior chance iff: (i) e is a macro-event, (ii) w is deterministic, (iii) $t_e > t_0$, and (iv) $0 < ch<p_e, w, t> < 1$.
>
> Deterministic *micro-initial* chance: ch$<p_e, w, t>$ is a deterministic micro-initial chance iff: (i) e is a micro-event, (ii) w is deterministic, (iii) $t_e = t_0$, and (iv) $0 < ch<p_e, w, t> < 1$.
>
> Deterministic *macro-initial* chance: ch$<p_e, w, t>$ is a deterministic macro-initial chance iff: (i) e is a macro-event, (ii) w is deterministic, (iii) $t_e = t_0$, and (iv) $0 < ch<p_e, w, t> < 1$.[13]

The defender of deterministic chance might accept the existence of any or all of these. The question is which she should take to exist, given her arguments (Section 2).

The first argument for deterministic chance—no-connection—would seem to allow for any and all of these four sorts of deterministic chance. The second argument—paradigm cases—is most naturally limited to deterministic macro-posterior chance, for it involves propositions concerning such events as heads landings. The third argument—nonreductionism—is explicitly limited to the two forms of deterministic macro-chance. While the fourth

[13] Since neither the micro/macro nor the initial/posterior distinction is intended to be exhaustive (as per notes 11 and 12), the resulting four conceptions of deterministic chance are nonexhaustive. So strictly speaking, there could be other conceptions of deterministic chance outside of these four. For instance, if there were deterministic chance that only pertained to propositions not about individual, momentary event occurrences (e.g. if the objective chance function at time t of deterministic world w outputted a 0.7 concerning $p : 2 + 2 = 4$ and all pigs have wings), it would be outside of these four conceptions.

Such a scenario is not really relevant to the current debate. I will suggest (in the main text below) that the question of whether there can be deterministic chance is best replaced by four questions, concerning whether there can be deterministic chance on each of the four conceptions. This replacement will have the added advantage of properly excluding such a scenario from the debate.

If there could be some other form of deterministic chance, I would regard that as showing something quite surprising about objective chance, namely, that the chances pertaining to individual, momentary event occurrences do not subvene the chances overall. But in any case, I would regard the question of whether there could be deterministic chance on some further conception, as a separate question (one concerning the interrelation between chances) not addressed in the main text.

argument—CSM explanations—has been explicitly linked by Loewer ([2001], [2004]) to deterministic macro-initial chance. This last linkage requires further discussion.

For Loewer, the case for deterministic chance is tied into a strategy for wedding Albert's ([2000]) treatment of CSM to Lewis's ([1973], [1994]) conception of lawhood. Albert's idea is that CSM requires the following to hold, as laws: (i) the Newtonian dynamical law: $F = ma$; (ii) the Past Hypothesis: the initial conditions are low entropy; and (iii) the Statistical Postulate: there is a probability distribution uniform on the standard measure over those regions of phase space compatible with our empirical information ([2000], p. 96).[14] Lewis's view is that the laws are the theorems of the best deductive systematization of the occurrent facts, where bestness is the optimal balance of simplicity, strength, and fit ([1994], Section 4). So Loewer's strategy is to argue that the Albert package of (i)–(iii) forms the Lewis laws for Newtonian worlds. Here, Loewer points out that the Albert package buys far greater strength than just the deterministic Newtonian dynamics of (i), since the Albert package 'entails probabilistic versions of all the principles of thermodynamics' ([2001], p. 618). Further, the Albert package costs just a bit of simplicity. So, adding (ii) and (iii) as law axioms seems worth the cost. (I will question this argument in Section 5. For now, I am only interested in what conception of deterministic chance it presupposes.)

Deterministic macro-initial chance is crucial to Loewer's strategy. The greater strength of the Albert package is generated by treating the Statistical Postulate as a chance-projecting axiom, since that is needed for the probabilistic thermodynamical laws to come out as theorems. Treating the Statistical Postulate as a chance-projecting axiom is recognizing an axiom projecting a deterministic macro-initial chance. The chance is initial since it concerns the initial conditions. The chance is macro since it concerns a coarse graining of the initial conditions.[15] Thus, Loewer is proposing to ground CSM in deterministic chances for the macro-initial conditions.[16]

14 Point of clarification: Albert ultimately recommends founding statistical mechanics on quantum mechanics as per the Ghirardi, Rimini, and Weber interpretations. Here, Albert notes that the chance-projecting Statistical Postulate is no longer needed, since: 'All the statistics there *are* in this theory ... are the purely *quantum-mechanical* ones in the GRW equations of motion' ([2000], p. 161). But this should not affect the argument that deterministic chance is at least possible, given that Newtonian worlds (as well as Bohmian worlds) remain at least possible.

15 The reason the chance can only concern a coarse graining of the initial conditions is because there are continuum-many possible micro-initial conditions, and applying equiprobability across continuum-many possible states requires assigning chance 0 to each state. The solution is to assign probabilities to macro-conditions, in the sense of *finite regions* of the phase space (Albert [2000], p. 63).

16 Since the incompatibilist will reject Loewer's deterministic macro-initial chances, it is worth asking how much of the Albert view of CSM she can accept. I think the answer is—*virtually all of it*. That is, the incompatibilist can accept (i) the Newtonian dynamical law (no problem with that!), (ii) the Past Hypothesis, and (iii) the Statistical Postulate (see footnote 31 for

These four conceptions of deterministic chance are worth distinguishing, because the arguments distinguish them. Thus the question of whether there can be deterministic chance is best replaced by four more specific questions:

1. Can there be deterministic micro-posterior chance?
2. Can there be deterministic macro-posterior chance?
3. Can there be deterministic micro-initial chance?
4. Can there be deterministic macro-initial chance?

The incompatibilist, then, must connect determinism to chance, on all four conceptions. This will require saying more about what chance is.

4 The Role of Chance

What is chance? So far I have characterized chance in a purely formal way, as a probability map from propositions, worlds, and times onto the closed unit interval (*Chance-formal*: Section 2). But there are many functions that fit this description. So the question becomes, which of these formally eligible functions is the chance function? To answer this question to describe the role of chance. For chance is as chance does. What renders a given formally eligible function fit to be the chance function is the extent to which it plays the chance role.

So what is the role of chance? Chance is connected to such further notions as *credence, possibility, futurity, intrinsicness, lawhood*, and *causation*. To characterize the role of chance is to trace such connections. What follows are some platitudes about these connections.

Starting with credence, information about chance should guide rational credence. Roughly, information about the objective chance of p should fix your credence in p onto its chance value. Lewis's ([1986]) Principal Principle (PP) provides a plausible way to be precise about this connection:

Principal Principle: $C(p/XE) = x$

Here C is any reasonable initial credence function, p is the target proposition, X is the proposition that $ch<p, w, t> = x$, and E is any admissible evidence.

further discussion). The only thing she cannot accept is the claim that the Statistical Postulate is a *law*. For her, the Statistical Postulate is merely a calculational crutch, useful for making predictions in ignorance of the details. Now it is true that Albert himself regards the Statistical Postulate as a law ([2000], p. 96), but this is only because he assumes that CSM is providing probabilistic explanations ([2000], p. 64) rather than probability of explanation. As far as I can see, nothing else in the Albert view requires that the Statistical Postulate (or Past Hypothesis) be a law. What emerges here, yet again, is that nothing is at stake concerning the foundations of CSM—all that is at stake is the philosophical interpretation, here whether (iii) should have the status of a *law* or not.

The PP says that, given the information that the chance of p at $<w, t>$ is x, plus any other admissible information at $<w, t>$, one's credence in p should lock onto x.[17]

Turning to possibility, nonzero chances should ground realizing possibilities. Roughly, if there is a nonzero chance of p, this should entail that p is possible, and indeed that p is compossible with the circumstances. Bigelow, Collins, and Pargetter's ([1993]) Basic Chance Principle (BCP) is a plausible way to be precise about this connection:

> *Basic Chance Principle*: If $ch<p, w, t> > 0$, then there exists a world w_{ground} such that: (i) p is true at w_{ground}, (ii) w_{ground} matches w in occurrent history up to t, (iii) $ch<p, w_{ground}, t> = ch<p, w, t>$.

The BCP says that, if the chance of p at $<w, t>$ is nonzero, then p is compossible with w up to t, in the sense that there is a possible world w_{ground} with the same history up to t and same chance for p, at which p is true. Here, w_{ground} is grounding the positive chance of p at w. Indeed, an even stronger principle is equally plausible:

> *Realization Principle* (RP): If $ch<p, w, t> > 0$, then there exists a world w_{ground} such that: (i) p is true at w_{ground}, (ii) w_{ground} matches w in occurrent history up to t, (iii) w_{ground} matches w in laws.

It is hard to imagine anyone accepting the BCP but rejecting the RP. For just as it might be said, in support of clause (iii) of the BCP, that: 'Worlds in which the present chance of [p] is different to the value it actually has are simply irrelevant when it comes to grounding the positive chance of [p] in the actual world' (Bigelow *et al.* [1993], p. 459), so the same may be said for the revised clause (iii) of the RP—to ground the positive chance in the actual world with the actual laws, the grounding world must keep the same laws.

Moving to futurity, chances should only concern future events. In general, chances should evolve through time in such a way that (i) they may fluctuate

[17] Lewis once claimed that the PP captures 'all we know about chance' ([1986], p. 86; though in discussion he considered retracting this claim, in light of the argument of Arntzenius and Hall [2003]). In any event, I think we know both less and more. We know less, in that we do not yet know the exact form that the chance-credence link should take. Chance guides rational credence, but it remains contentious as to how to be precise about this. The PP is plausible but not sacrosanct. Yet we know more, in that in the very same way that we know that chance guides rational credence, we also know other theoretical platitudes about what chance does—chance grounds real possibility, it applies only to the open future, it takes the same values for intrinsically duplicate trials, it is projected by the laws, and it is involved in causation. These sorts of platitudes have the same status as the chance-credence link. In all these cases, it is intuitively obvious that some conceptual linkage exists, and in all of these cases it is philosophically difficult to articulate the linkage precisely. What we know about, generally, are a range of theoretical platitudes about the chance role. Nothing special about chance here: this is what we know about the meanings of theoretical terms generally.

up to the target time t_e, but (ii) at and after the target time t_e, they are fixed at 1 or 0, depending on whether e in fact occurred or not. The intuitive rationale is that the future is *open*, but the past and present are *fixed* (Bigelow *et al.* [1993], p. 454). This idea is well-illustrated by Lewis's example of the labyrinth:

> Suppose you enter a labyrinth at 11:00 a.m., planning to choose your turn whenever you come to a branch point by tossing a coin. When you enter at 11:00, you may have a 42% chance of reaching the center by noon. But in the first half hour you may stray into a region from which it is hard to reach the center, so that by 11:30 your chance of reaching the center by noon has fallen to 26%. But then you turn lucky; by 11:45 you are not far from the center and your chance of reaching it by noon is 78%. At 11:49 you reach the center; then and forevermore your chance of reaching it by noon is 100%. ([1986], p. 91)

With this in mind, I would propose the following Future Principle (FP):

> *Future Principle*: If $0 < ch<p_e, w, t> < 1$, then $t < t_e$.

The FP says that, if the chance of p_e at $<t, w>$ is between 0 and 1, then p_e must concern t's future. To violate the FP is to undo the connections between what is chancy and what is still open.[18]

Continuing to intrinsicness, chance values should remain constant across intrinsically duplicate trials. The intuitive rationale for this is that if you repeat an experiment, the chances should stay the same. For instance, if the chance that the first coin flip lands heads is 0.5, and the second coin flip represents an intrinsically duplicate trial (exactly the same sort of coin, flipping, and environment), then the chance that the second coin flip lands heads should also be 0.5. Thus I would propose the following Intrinsicness Requirement (IR):

> *Intrinsicness Requirement*: If e' is an intrinsic duplicate of e, and the mereological sum of the events at t' is an intrinsic duplicate of the mereological sum of the events at t, then $ch<p_e, w, t> = ch<p_{e'}, w, t'>$.

To violate the IR is to undo the connections between the magnitude of chance and the intrinsic features of the trial.[19]

[18] As formulated, the FP presupposes some foliation of space-time that defines 'past, present, and future', and so determines whether or not $t < t_e$. Here scientific knowledge may require some conceptual revision. I lack the space to discuss the details further.

[19] The IR is a variant of what I have elsewhere called the *Stable Trial Principle* (STP): 'If (i) A concerns the outcome of an experimental setup E at t, and (ii) B concerns the same outcome of a perfect repetition of E at a later time t', then $P_{tw}(A) = x = P_{t'w}(B)$' ([2003], p. 37). Note that both the STP and the IR are formally restricted to intra-world duplicates—duplicates at different worlds might have different chances in virtue of differences in what the laws are. Note

Shifting to lawhood, chance values should fit with the values projected by the laws of nature. For instance, if the chance that the coin lands heads is 0.5, then the laws should codify that value (via history-to-chance conditionals). This suggests the following Lawful Magnitude Principle (LMP):

> *Lawful Magnitude Principle*: If $ch\langle p, w, t\rangle = x$, then the laws of w entail a history-to-chance conditional of the form: if the occurrent history of w through t is H, then $ch\langle p, w, t\rangle = x$.

The LMP connects the magnitude of chance to the magnitude of a lawfully projected quantity. To violate the LMP is to sever chance from lawhood.[20]

Finishing with causation, chances should live within the causal transitions they impact. That is, if a given chance is to explain the transition from cause to effect, that chance must concern some event targeted within the time interval from when the cause occurs, to when the effect occurs. Otherwise that chance cannot impact the transition from cause into effect—it would be left outside of the action. This may be codified as the Causal Transition Constraint (CTC):

> *Causal Transition Constraint*: If $\text{ch}\langle p_e, w, t\rangle$ plays a role in the causal relation between c and d, then $t_e \in [t_c, t_d]$.

The CTC requires that causal chances arise within the causal intervals they impact. To violate the CTC is to posit chances that cannot reach the causal action.

Putting this together, I am now in a position to identify substantive constraints on chance. Chance is the function that plays the role of chance in credence, possibility, futurity, intrinsicness, lawhood, and causation:

> *Chance-substantive*: Chance is what best satisfies (i) the Principal Principle, (ii) the Realization Principle, (iii) the Futurity Principle, (iv) the Intrinsicness Requirement, (v) the Lawful Magnitude Principle, and (vi) the Causal Transition Constraint.

Combining *Chance-substantive* with *Chance-formal* produces:

> *Chance*: Chance is that probability function from propositions, worlds, and times onto the closed unit interval, which best satisfies: (i) the Principal Principle, (ii) the Realization Principle, (iii) the Futurity Principle, (iv) the Intrinsicness Requirement, (v) the Lawful Magnitude Principle, and (vi) the Causal Transition Constraint.[21]

also, that both the STP and the IR should properly be supplemented with some sort of 'causal independence of trials' proviso.

[20] For further discussion of the LMP, see (Schaffer [2003]; Lange [2006]).

[21] Given that the requirements of *Chance-formal* fall out of the Principal Principle (Lewis [1986]), *Chance* might seem to fall out of *Chance-substantive*. Though there is the following difference.

For present purposes, *Chance* will stand as my final account of what chance is.[22]

It does not matter for present purposes whether *Chance* is sufficiently constraining so as to characterize chance *uniquely*. Perhaps there remain multiple best satisfiers of *Chance*. Even if so, I will be arguing that no deterministic chance function is among them.

Nor does it matter if one or two of the constraints in *Chance* are best abandoned. I think they are all broadly platitudinous. At least, they have generally been taken to be such in the literature so far (not for nothing have they been given names like 'The Principal Principle' and 'The Basic Chance Principle'). But perhaps a few of these constraints will prove contentious.[23] Either way, I will be arguing that no deterministic chance function can even fit the majority of the constraints. That is, I am about to argue that none of the four conceptions of deterministic chance is more than *half-eligible* to play the role of chance.

Of course, even being half-eligible can be good enough, if there are no better candidates. But I am also about to argue that there is a perfect candidate to play the role of chance—the incompatibilist function that outputs just 0s and 1s in deterministic worlds.

5 The Case against Posterior Deterministic Chance

I am now in a position to connect determinism and chance, by arguing that none of the four conceptions of deterministic chance can play the chance role. In this section, I will consider deterministic micro- and macro-posterior chance. In Section 6, I will consider deterministic micro- and macro-initial chance.[24]

Chance-substantive alone allows that the chance function might not be a probability function, since the best satisfier of its constraints (i)–(vi) might still violate (i). While *Chance* requires that the chance function be a probability function, and only searches for best satisfiers in the space of such functions.

22 For related characterizations of the role of chance, see (Loewer [2001]; Schaffer [2003]). Though the reader should beware that Loewer builds in two constraints that I am in the process of rejecting, namely: 'Chance distributions over the initial conditions of the universe make sense,' and 'Chances different from 0 and 1 are compatible with determinism' ([2001], p. 613).

23 One might question whether these constraints fit a Humean picture, according to which chance values supervene on occurrent history. I take no stand on Humeanism here. By my lights, the question of whether these constraints fit a Humean view of chance *is* the question of whether a Humean view of chance is defensible. In this vein, see (Schaffer [2003]) for a Humean account that fits the PP, RP, FP, and CTC perfectly, and at least approximates the LMP and IR.

24 The micro/macro distinction will turn out not to play an important role in the arguments that follow. Its interest lay mainly in distinguishing where the four arguments for deterministic chance lead (Section 3), since many of them (paradigm cases, nonreductionism, CSM explanations) were explicitly limited to macro-chances. In what follows, the initial/posterior distinction will prove more important.

Starting with deterministic micro-posterior chance, such a conception of chance cannot fit the Principal Principle, Realization Principle, or Lawful Magnitude Principle. As to the PP, suppose that ch$<p_e, w, t>$ is a deterministic micro-posterior chance, where e in fact occurs. Since ch$<p_e, w, t>$ is a deterministic micro-posterior chance, w is deterministic and ch$<p_e, w, t> < 1$. By *Determinism*, the laws of w plus the state of w at t fix the occurrence of e. The laws of nature are admissible, as is the state of the world at t, so the admissible evidence E entails p_e.[25] That entails that, for any reasonable credence function C, C(p_e/E) = 1. This in turn entails that, for any X compatible with E, C(p_e/XE) = 1, so by the PP, ch$<p_e, w, t> = 1$. Now ch$<p_e, w, t> < 1$ and ch$<p_e, w, t> = 1$. Contradiction. Conclusion: the only micro-posterior chances that the PP allows in deterministic worlds are 0s and 1s. So only an incompatibilist function—one that outputs just 0s and 1s in deterministic worlds—can fit the PP.

I can only hope that the compatibilist has money, and is willing to take bets that are fair by her lights. For if she and I both know that e is determined to occur, and yet she evaluates ch$<p_e, w_{actual}, t>$ as less than 1, then I will win all her money. For she will take bets against p_e with the following payout scheme, for arbitrary sum n:

If e does not occur: The compatibilist wins $n(1-x)$

If e does occur: The compatibilist loses $0.5(nx)$

By the compatibilist's lights, this is not just fair but biased in her favor. For, given the PP and her view of chances, her expectation is: $(xn(1-x)) - ((1-x)(0.5(nx)))$, which is positive for $0<x<1$ and $n>0$.[26] She will expect to win money. She will lose everything.[27]

I anticipate two responses. First, one might deny that the laws of nature are admissible. After all, such a respondent might say, did we not learn from the *undermining problem* (Lewis [1994]) that the laws of nature contain inadmissible information about the future? But first, the laws of nature had better be admissible generally, or the PP is worthless. At least, the intuitions behind the PP presuppose the admissibility of the laws (Lewis [1986], pp. 130–1). And second, none of the solutions to undermining help the compatibilist. Whether the undermining problem is solved by (i) replacing the PP with the

[25] Here I am assuming that any reasonable initial credence function C assigns 0s to metaphysical impossibilities. This assumption can be waived, by conditionalizing the credences on determinism. Alternatively, this assumption can be relaxed by including in E the evidence so far that w is deterministic. Now E will not entail p_e, but it will presumably boost rational credence in p_e past ch$<p_e, w, t>$. This will suffice for a contradiction. For instance, the evidence so far might be that there have been a thousand coin-flips in these same micro-conditions, and all have landed heads. This will not boost p_{heads} to 1, but it will certainly boost it past 0.5. This will suffice for the contradiction of ch$<p_e, w, t> = 0.5$ and ch$<p_e, w, t> > 0.5$.

New Principle (NP), on which credences go conditional on the theory of chance (Thau [1994]; Hall [1994]; Lewis [1994]); (ii) redefining chances so that the chance values themselves are the Lewis-chances conditional on the theory of chance (Arntzenius and Hall [2003]; Schaffer [2003]); or (iii) abandoning Humean Supervenience (Bigelow *et al.* [1993]; *inter alia*), the problem will recur. Indeed, the only solution that even touches the argument is (i) the shift to the NP, but the quantities outputted by the PP and the NP should be so similar that the compatibilist would still suffer the fate of the sucker, by endorsing bets she knows she'll lose.[28]

Second, one might revise the PP. Perhaps one can replace E with E*, where E* does not admit the laws of nature and/or the state of the world at t. For instance, (Loewer [2001], p. 618) offers the following revision:

$$PP_{macro}\text{: } C(p/XE_{macro}) = x$$

Here E_{macro} does not allow 'macroscopically inadmissible information,' such as the micro-state of the world at t. But this looks like surrender. For what connects credence to chance is still (something like) the PP. Showing that deterministic micro-posterior chance fits *some other principle* is irrelevant (and calling it 'PP_{macro}' cannot help). Or at least, the revisionist will need to explain why her revised principle bears any real connection to objective chance, especially when setting one's credences to it will make one endorse bets one knows are doomed.

26 The simplest way to see this is to note that $xn(1-x) = (1-x)nx$ where both are positive, and then to note that the payout halves the risk. To illustrate, if the compatibilist thinks that the chance of e is $x = 0.9$, and we set n to 1, she will evaluate her reward at:

$$xn(1-x) = 0.9 \times 0.9 = 0.81 \tag{1}$$

While she will evaluate her risk at:

$$(1-x)(0.5(nx)) = 0.9 \times 0.5 \times 0.9 = 0.405 \tag{2}$$

So the compatibilist's expected value is positive. So she should regard such a bet as favorably biased—indeed, she should happily ante anything up to 0.405 for the action!

27 Of course the compatibilist might not be a gambler, or care about keeping her money. That is beside the point. The point of the betting argument is to illustrate her *irrationality*, even if she remains prudish enough not to bet, or stoic enough not to care.

28 Hoefer [unpublished] offers a more sophisticated version of this sort of reply. Hoefer argues that E is admissible relative to p iff: the only information E carries as to whether p, is information about the objective chance of p. From this he concludes that given determinism, the laws and the state at t cannot be *jointly* admissible. But this seems to me to be too limited a conception of what is admissible. For on this conception, some facts that are purely about the past and present will count as inadmissible. Thus, suppose that an unbiased coin is flipped at $<w, t>$, with an alleged 0.5 chance of landing heads. And suppose that so far there have been a million coin flips at w done under exact duplicate micro-conditions, every one of which has landed tails (perhaps there have also been other coin flips at w under different micro-conditions, with differing results). This past frequency data seems to carry information that our present flip will almost certainly land tails, and so would be inadmissible by Hoefer's lights. But surely, past frequency data is admissible if anything is.

As to the RP, deterministic micro-posterior chance will fare no better. So suppose that $\mathrm{ch}<p_e, w, t>$ is a deterministic micro-posterior chance, where e in fact does not occur. Then $\mathrm{ch}<p_e, w, t> > 0$ and w is deterministic. Since $\mathrm{ch}<p_e, w, t> > 0$, the RP entails that there is a world w_{ground} such that: (i) p_e is true at w_{ground}, (ii) w_{ground} matches w up to t, (iii) the laws of w_{ground} match the laws of w. But since w is deterministic, w_{ground} cannot match w up to t and have the same laws as w, without matching w throughout all of history, including the nonoccurrence of e.[29] So e occurs at w_{ground} and e does not occur at w_{ground}. Contradiction. Conclusion: the only micro-posterior chances that the RP allows in deterministic worlds are 0s and 1s. If the one outcome is determined to occur, the other cannot possibly be realized. So only an incompatibilist function can fit the RP.

As to the LMP, deterministic micro-posterior chance will fail as well. So suppose that $ch<p_e, w, t>$ is a deterministic micro-posterior chance. Then the LMP entails that there is a lawfully entailed history-to-chance conditional of the form: if the history of w through t is H, then $ch<p, w, t> = x$ (where $0<x<1$). But laws at deterministic worlds do not project chances. For instance, if the laws are Newtonian laws, then they do not project chance values at all (except for the extremal 0s and 1s). There is no chance parameter hidden in $F = ma$. The Newtonian laws entail history-to-occurrence conditionals only.

I anticipate the response that the macro-laws at deterministic worlds may still project chances. Indeed, such a respondent might say, doesn't Loewer's argument (from Section 3) that the Albert package forms the Lewis laws show exactly how such macro-lawful chances might arise? But there are two problems with Loewer's argument. The first problem is that the Lewis view explicitly restricts the eligible candidates for best systemhood, to those candidates whose predicates refer only to perfectly natural properties (Lewis [1983], pp. 367–8).[30] The Albert package contains predicates such as 'low entropy' that refer to properties that are not perfectly natural—in microphysical vocabulary, that property is infinitely disjunctive. Hence the Albert package is *not even in the running* for the Lewis laws. It is ineligible from the start.

The second problem with Loewer's argument is that, even if the Albert package were in the running, it would still face further competitors. Loewer compares (i) $F = ma$; to the Albert package of (i) with (ii) the Past Hypothesis, and (iii) the Statistical Postulate. I agree that (i)–(iii) is a better package than (i) alone—indeed (i) alone entails little about the occurrences. But there is

[29] Posteriority is playing a role here, in ensuring that there exists a time prior to t at which w' matches w.

[30] The restriction is needed to prevent the best systems account from collapsing into vacuity. For if predicates referring to any property were allowed into the competition, then any decent system S would be bettered by the vacuous system whose sole axiom is: $(\forall x)Fx$, where F applies to all and only the things at worlds where S holds (Lewis [1983], pp. 367–8)

another competitor to consider, which is the package of (i) with (iv) the Precise Initial Conditions.[31] This package is roughly equal in simplicity to (i)–(iii)—one complex condition in (iv) versus two simpler ones in (ii) and (iii). But the package of (i) + (iv) is vastly stronger than (i)–(iii), since (i) + (iv) entails every single detail of the entire history of the world. So (i)–(iii) will not count as the Lewis laws, after all—(i) and (iv) provides an even better systemization.[32]

Perhaps there are other ways (other than Loewer's) to allow that the macro-laws at deterministic worlds may still project chances.[33] But for present purposes it should suffice to note that (i) if w is deterministic, and the history of w at the initial moment t_0 is H, then the laws of w fix the history of

[31] This seems to be the sort of package that Lewis himself had in mind: 'The ideal system need not consist entirely of regularities; particular facts may gain entry if they contribute enough to collective simplicity and strength. (For instance, certain particular facts about the Big Bang may be strong candidates)' ([1983], p. 367). Though there is the worry, mentioned to me by Loewer as well as Adam Elga, that the Precise Initial Conditions may prove infinitely complex. Here things depend on (i) the empirical details, as well as (ii) what predicates are eligible. If Loewer can use predicates such as 'low entropy' to secure reference to what would (in microphysical vocabulary) be an infinite disjunction, there seems no barrier to using the phrase 'actual initial conditions' to refer to what might otherwise be an infinite conjunction. While if the empirical details and considerations of eligibility disqualify the Precise Initial Conditions, one might still consider various systems packaging $F = ma$ with ever more perfectly natural information about the Big Bang. Such systems will presumably trade simplicity for strength, as more information about the Big Bang is added. If nature is kind, there will be an optimal balance point. That will determine the best system.

[32] So if (i) and (iv) are the law axioms, can Albert's view of CSM be recovered? *Yes*, the Past Hypothesis and Statistical Postulate are recoverable as follows. The Past Hypothesis is recovered from (iv) plus bridge principles connecting micro-states to entropy values. That is, given the Precise Initial Conditions and the appropriate bridge principles, the Past Hypothesis (if true) will be a theorem. The Statistical Postulate is trickier to recover, because one must justify the use of the standard probability measure (Albert [2000], pp. 63–4; Loewer [2001], p. 615). Here I would not simply invoke indifference, since there is no unique way to apply indifference by itself (van Fraassen [1989], ch. 12). Rather I would invoke indifference *plus* the objective naturalness of the standard measure. What I mean by invoking the objective naturalness of the standard measure, is that occupying such-and-such surface area of an energy hypersurface is a decently natural macro-property. How that helps is to constrain the application of indifference, by letting nature dictate the measure (see Section 7 for further discussion). Putting this all together, from (i) and (iv), together with (a) bridge principles connecting micro-state to entropy values, and (b) the metaphysical posit of the naturalness of the standard measure, everything Albert claims is needed for CSM can be had—without any terribly revisionary treatment of chance.

[33] For instance, one referee suggested that some macro-laws may be essentially regional, consequences of the micro-laws plus pervasive local conditions (e.g., the 'law' that acceleration due to gravity at the surface of the earth is 9.8 ms^{-2}). So perhaps the local conditions could make room for chances, somehow. Though to my mind the local conditions only make for *ceteris paribus* (all else equal) conditions—the law about acceleration due to gravity at the surface of the earth is that such acceleration is at the rate of 9.8 ms^{-2}, *ceteris paribus*. Whether all else is equal in a given case is fixed by the associated micro-conditions, though we are typically ignorant of what exactly these micro-conditions are, and whether or not these count as normal with respect to the relevant macro-law. Obviously this and related issues cannot be settled without a full discussion of lawhood, the relation between micro and macro-laws, and the relation between strict and *ceteris paribus* laws—which is well beyond the scope of the current discussion.

w after t_0 to $H+$ (by *Determinism*), so (ii) the laws of w seem to suffice for (nonchancy, maximally strong) history-to-occurrence conditionals of the form: if the history of w through t_0 is H, then the history of w after t_0 is $H+$. Here $H+$ covers everything—it represents the full and precise posterior evolution of w. So, given the deterministic dynamics and the initial conditions, the system is already maximally strong. Any extra 'laws' projecting micro-posterior chances prove needless. Conclusion: the only micro-posterior chances that the LMP allows in deterministic worlds are 0s and 1s. If the one outcome is determined to occur, the laws will project that it will occur. So only an incompatibilist function can fit the LMP.

What emerges is that deterministic micro-posterior chance is ineligible to serve as genuine chance. While it may fit the FP, IR, and CTC (I see no problem there), it cannot fit the PP, RP, or LMP. This is hardly a viable conception of chance, especially since it must compete with the incompatibilist conception, which fits every platitude perfectly. Thus the no-connection argument (Section 2), which allows for deterministic micro-posterior chance (Section 3), fails once *Chance-substantive* is integrated into *Chance-formal* to yield *Chance* (Section 4). The PP, RP, and LMP impose substantive connections between the supervenience thesis of *Determinism* and the sort of probability of function that can satisfy *Chance*.

Now, deterministic macro-posterior chance fares no better than deterministic micro-posterior chance (which is why I have grouped them together). As to the PP, the compatibilist will still be committed to chances over what is determined, and hence committed to endorsing bets she knows she'll lose. As to the RP, the compatibilist will still be committed to chances that cannot possibly be realized, given how the world is. As to the LMP, given that nomological supervenience correlates chances (Section 2), the lawfully entailed history-to-chance conditionals for propositions concerning macro-events must correlate with the lawfully entailed history-to-chance conditionals for propositions concerning their bases. Since the latter will project only 0s and 1s, the former will also project only 0s (if all of the basis-propositions get chance 0) or 1s (if any basis-proposition gets chance 1).

What emerges is that the paradigm case argument—which pushed for deterministic macro-posterior chance (Section 3)—is pushing against the PP, RP, and LMP. To insist that a heads landing retains an objective 0.5 chance when the coin is determined to land tails, is to sever the connections from chance to credence, possibility, and lawhood. For one should not set one's credence to 0.5 given all the admissible information, the 0.5 chance is not compossible with how things really are, and the 0.5 chance is not projected by the laws. This provides further evidence that the sort of chance involved in deterministic coin flips is merely epistemic.

Similar comments apply to deterministic thermodynamical chance. To insist that the ice cube retains an objective though miniscule chance of not melting in the cup of warm water when it is determined to melt, is also to sever the connections from chance to credence, possibility, and lawhood. For one should not set any credence in melting below 1 given all the admissible evidence, the sub-1 chance is not compossible with how things really are, and the sub-1 chance is not projected by the laws. This provides further evidence that the sort of chance involved in CSM is merely epistemic.

6 The Case against Initial Deterministic Chance

To finish connecting *Determinism* and *Chance*, it remains to consider the prospects for deterministic micro-initial and macro-initial chance. Starting with deterministic micro-initial chance, such a conception of chance cannot fit the Principal Principle, Lawful Magnitude Principle, Futurity Principle, Intrinsicness Constraint, or Causal Transition Constraint. As to the PP, the problem is that at any time of assessment, the full information about the initial conditions will be admissible. So E entails $p_{initial}$, so $C(p_{initial}/E) = 1$. Thus, for any X compatible with E, $C(p_{initial}/XE) = 1$, so $x = 1$. In general, at any time of assessment, given the full admissible information, one's credence in the Precise Initial Conditions (and what they entail) should be 1, and one's credence in any rival hypothesis about the initial conditions should be 0. Only an incompatibilist function can rationalize the right credences.[34]

As to the LMP, the problem is also essentially the same as with deterministic prior chance (Section 5). The laws at, e.g. Newtonian worlds, do not project chances. (Here I am assuming that the best system for Newtonian worlds is formed by packaging the Newtonian dynamic laws with the Precise Initial Conditions: Section 5.)

As to the FP, suppose that $ch<p_e, w, t>$ is a deterministic micro-initial chance. Then $t_e = t_0$. But then, since there is no time before t_0, the time of assessment must be at or after t_e: $t \geqslant t_e$. This is in direct violation of the FP, which requires $t < t_e$ here. Since there is no time before t_0, the $ch<p_e, w, t>$ must constitute a chance for the fixed past (if $t>t_0$) or the fixed present (if $t = t_0$). There simply is no time of assessment from which the initial conditions can count as open.

I anticipate that the FP may be dismissed as question-begging. But the FP has independent intuitive support, in the connection from chanciness to openness, and from openness to futurity. And the FP has further intuitive

34 Here, one does not even need to assume that the laws are admissible—it suffices that the past and present are admissible. So in that sense the problem with the PP for initial chances may be even worse than for posterior chances.

support from examples like Lewis's labyrinth (Section 4). It would be bizarre to imagine that at 11:49 you reach the center of the labyrinth, but that at 11:49 the chance of your reaching the center is only, say, 0.75. If you got there, you got there; if not, not. Only an incompatibilist function treats the initial conditions in like manner.

As to the IR, suppose that ch$<p_{BigBang}, w, t>$ is a deterministic micro-initial chance, and suppose that w is a one-way eternal recurrence world. That is, w begins with a bang at t_0, and then oscillates to infinity, cycling through every state endlessly. Since ch$<p_{BigBang}, w, t>$ is a deterministic micro-initial chance, $0 < ch<p_{BigBang}, w, t> < 1$ and $t_{BigBang} = t_0$. Since w involves eternal recurrence, infinitely many intrinsic duplicates of the initial conditions will exist (at the turning point of each historical cycle, as the universe crunches and re-bangs). Let *Rebound* be one such re-banging. Then by hypothesis, *Rebound* is an intrinsic duplicate of *BigBang*, and the sum of events at $t_{Rebound}$ is an intrinsic duplicate of the sum of events at $t_{BigBang}$, so it follows from the IR that $ch<p_{Rebound}, w, t_{Rebound}> = ch<p_{BigBang}, w, t_{BigBang}>$. But $ch<p_{Rebound}, w, t_{Rebound}>$ is a deterministic micro-*posterior* chance. By the argument of Section 5, there is no such thing.

What emerges here is that the compatibilist cannot limit the chances to initial conditions, because of the possibility that such conditions get duplicated later. 'Initial condition' is an extrinsic characterization. In one-way eternal recurrence worlds in which such conditions are duplicated, the compatibilist must either (i) retreat to posterior chances, with a consequent violation of the PP, RP, and LMP; or (ii) violate the IR.[35]

Finally, as to the CTC, suppose that ch$<p_e, w, t>$ is a deterministic micro-initial chance. Then $t_e = t_0$. Now suppose that ch$<p_e, w, t>$ plays a role in the causal relation between c: the event of the ice cube entering the cup of warm water, and d: the event of the ice cube melting. By the CTC, for ch$<p_e, w, t>$ to play a role in the melting, t_e must be in the interval $[t_c, t_d]$. But it is not so—this initial chance is stuck at t_0, way back at the beginning of the universe, and so it remains outside the causal action. It cannot reach the ice.

By way of analogy, imagine that $w_{one\text{-}shot}$ has just one chancy event—a coin flip at t_i, say—which lands heads. And now consider some later time t_j at $w_{one\text{-}shot}$. Is there anything chancy left at $<w_{one\text{-}shot}, t_j>$? *No*: because

[35] The fact that the problem arises for one-way eternal recurrence worlds should not matter. It still seems wrong to think that it *must* fail at one-way eternal recurrence worlds. The IR should hold whether or not there is such recurrence. This problem, by the way, is distinct from the complaint that, because the initial conditions are by definition a one-shot affair, there are no measurable frequencies involved (Callender [2004], pp. 197–8). That is an epistemic complaint. The problem with the IR here is not that there are no measurable frequencies for initial conditions—it is rather that in one-way eternal recurrence worlds, there *are* measurable frequencies in the reference class of intrinsic duplicates, and the frequency of p there is precisely 1.

there is just one chancy event in $w_{one\text{-}shot}$, and it has already happened, and so its outcome is already fixed. The chances at $w_{one\text{-}shot}$ are spent. Likewise, deterministic micro-initial chances are spent from the moment the world begins. By the time the ice hits the cup of warm water, such chances are already spent.

I anticipate that the CTC may be accommodated by introducing a new sort of posterior chance. In this vein, Loewer offers the following construction:

> [W]e can define a kind of dynamical chance which I call 'macroscopic chance'. The macroscopic chance at t of event A is the probability given by starting with the micro-canonical distribution over the initial conditions, and then conditionalizing on the entire macroscopic history of the world (including the low entropy postulate) up until t. ([2001], p. 618)

While this is very clever, I have no idea why it deserves to be considered a chance function (and calling it 'macroscopic chance' cannot help). Not only does it not fit the PP, RP, or LMP (as per Section 5), but its limitations on what can be conditionalized on seem *ad hoc*. Ontologically, the chance is spent. That we can restrict what can be conditionalized on to hide this fact seems beside the point. For instance, one could concoct a function that would not allow for conditionalizing on the flip outcome at later times in $w_{one\text{-}shot}$. But that would hardly show that $w_{one\text{-}shot}$ continues to enjoy objective chances, after its one shot.

What emerges is that deterministic micro-initial chance is not even close to being eligible to serve as genuine chance. It cannot fit the PP, LMP, FP, IR, or CTC. The only constraint it fits is the RP. (Indeed, the only way it fits the RP is vacuously—since there is no time before t_0, clause (ii) of the RP, which requires that w' and w match up to t, goes vacuous, so the RP loses any bite.) In sum, this is hardly a viable conception of chance, especially since it must compete with the incompatibilist conception, which continues to fit every platitude perfectly.

Now deterministic macro-initial chance fares no better than deterministic micro-initial chance (which is why I have grouped them together). Indeed, exactly the same arguments with respect to the PP, LMP, FP, IR, and CTC will apply. As to the PP, the compatibilist will still be committed to chances over outcomes already admissible. As to the LMP, she will still postulate chances not projected by the laws (assuming the axioms at, e.g. Newtonian worlds, are the Newtonian dynamic law and the Precise Initial Conditions). As to the FP, she will still countenance chances over the fixed present or past. As to the IR, she will still be committed to chances that would be different under intrinsic duplicate trials in one-way eternal recurrence worlds. And as to the CTC, she will still be positing chances that are causally inert, because they are spent from the moment the world begins.

What emerges now is that the CSM explanation argument—which pushed for deterministic macro-initial chance (Section 3)—is pushing against the CTC. For the argument there was that mere epistemic chance could not help explain the melting of an ice cube (Section 2). But mere initial condition chance, spent long before the ice cube even exists, and not pertaining to any time at which the ice cube melts, cannot help explain the melting of an ice cube either. There just is no probabilistic explanation in the offing here. What explains the melting of the ice cube is the complex deterministic process that runs from the ice cube's entering the water to its melting, whose myriad details we can only guess. The only probability involved in the deterministic process of an ice cube melting is the measure of our ignorance of the real micro-explanation.

7 Epistemic Chance

So far, I have argued that there cannot be a deterministic chance on any of the four conceptions (Section 3), since the platitudes about the role of chance do not allow it (Sections 4–6). I have suggested that the sort of chance found in deterministic coin flips and in CSM is merely epistemic chance, objectively informed (Section 2). It remains to justify the distinction I have been presupposing between objective and epistemic chance, and perhaps explain where the compatibilist went wrong.

To begin with, here are two factors that do *not* distinguish objective from epistemic chance. One nondistinguishing factor is the bearing of objective information. Obviously, objective information can bear on objective chance. But objective information can also bear on epistemic chance, in at least two ways. First, there are objective constraints on *what to be indifferent about*. Compare, for instance, (i) the partial function $f1$ that maps $<$the first card drawn from this well-shuffled deck will be the ace of spades, $w, t_{shuffled}>$ to 1/52; with (ii) the partial function $f2$ that maps $<$the first card drawn from this well-shuffled deck will be the ace of spades, $w, t_{shuffled}>$ to 1/2. Here $f1$ is indifferent as to which card will be drawn, while $f2$ is indifferent as to whether or not the ace of spades will be drawn. Each can fit into a nonextremal overall probability function. But clearly it is $f1$ that has an objective basis. If we were to run experiments on well-shuffled decks, the frequency data would tend to confirm $f1$ and disconfirm $f2$. If all one knows is that one is drawing the top card from a well-shuffled deck, then one ought to use $f1$ for prediction rather than $f2$. If one wants to compute the *general* probability of drawing the ace of spades first, one ought to employ a function that embeds $f1$.[36]

[36] Similar comments will apply to partial functions that are indifferent over (i) heads or tails, versus heads-aligned-north, heads-aligned-south, or tails, and (ii) the standard CSM measure of

Second, objective information can guide epistemic chance updating, by guiding how we re-weight the cells that the initial indifference measure isolates. For instance, information about the details of the shuffle can update the epistemic chance of drawing the ace of spades first, information about the mass-density distribution on the coin can re-weight the epistemic chance of heads versus tails, and information about the micro-distribution of particles can impact the epistemic chance of occupying a certain surface area of an energy hypersurface.

It should not be surprising that objective information can guide epistemic chance. Information impacts what we know.

A second factor that does *not* distinguish objective from epistemic chance is having some use in the sciences. Objective chance is invoked in quantum mechanical wave collapse (at least on the Copenhagen and GRW interpretations). To my mind, this is part of what was so radical about quantum mechanics—here is where objective chance first entered the scientific worldview.

Epistemic chance is used in sciences like classical genetics. For instance, the Mendelian will invoke chances to predict the genotype resulting from a dihybrid cross, and will rule that the chance that a common garden pea (*pisum sativum*) will have both green and dented seeds, at w, at the time of crossing t_{cross}, is 1/16. She will even formulate laws to the effect that the probability of a dihybrid cross with two heterozygous parents yielding a doubly recessive genotype in the next generation is 1/16.[37]

It should not be surprising that epistemic chance has a use in the sciences. What we know impacts to what extent we can make predictions.

So what does distinguish objective from epistemic chance? What distinguishes them is the objective chance role (Section 4). It takes more than being objectively informed and wearing scientific credentials, for a probability function to be an objective chance. To be an objective chance, a probability function must also fit the connections from objective chance to credence, possibility, time, intrinsicness, lawhood, and causation. These platitudes thus not only connect *Determinism* to *Chance*, they also ground the objective/epistemic distinction.

I suspect that here may be where the compatibilist went wrong—she saw an objectively informed chance with scientific credentials, and supposed that it must be an objective chance. Objectively informed and scientifically credentialed chances are fully compatible with determinism, as we already knew from Mendelian genetics and Gibbsian statistical mechanics. But

occupying such-and-such surface area of an energy hypersurface, versus some gerrymandered measure.

37 This follows from Mendel's Third Law (the Law of Independent Assortment), and may be illustrated by the method of Punnett squares.

they are still not objective chances, as the platitudes reveal.[38] There is an ontological difference between the sort of chances invoked in Mendelian genetics (epistemic) and those invoked in quantum mechanical wave collapse (objective), which the compatibilist would collapse.

In this vein, imagine a particular well-shuffled deck at <w, $t_{shuffled}$>, with the three of hearts sitting on top of the deck. If one had all the present information, one should set one's credence for <the first card drawn from this well-shuffled deck will be the three of hearts, w, $t_{shuffled}$> to 1, and set one's credence for <the first card drawn from this well-shuffled deck will be the ace of spades, w, $t_{shuffled}$ > to 0. Given the state of w at $t_{shuffled}$ and the laws, it is not possible that the ace of spades (stuck in the middle of the deck) be drawn first. The deterministic laws of w support conditionals of the form: if the history of w through $t_{shuffled}$ is H, then the three of hearts will definitely be drawn. This is why no function that maps <the first card drawn from this well-shuffled deck will be the ace of spades, w, $t_{shuffled}$ > to anything other than 0 can be an objective chance function. Rather, it is a measure of our ignorance as to which card is in fact on top.

Or imagine a particular common garden pea at <w, t_{cross}>. The seeds are in fact genetically programmed to be, say, yellow and smooth. So if one had all the present information, one would be rational to set one's credence for <this common garden pea will have both yellow and smooth seeds, w, t_{cross}> to 1, and to set one's credence for <this common garden pea will have both green and dented seeds, w, t_{cross}> to 0. Given the state of w at t_{cross} and the laws, it is not possible that green and dented seeds will result. The deterministic laws of w support conditionals of the form: if the history of w through t_{cross} is H, then yellow and smooth seeds will definitely result. This is why no function that maps <this common garden pea will have both green and dented seeds, w, t_{cross}> to anything other than 0 can be a chance function. Rather, it is a measure of our ignorance as to which genes were in fact passed on. Ontologically, the case of the genotype of the pea is no different from the case of the shuffled deck of cards.

What goes for genotypes and shuffled cards goes equally for coin flips and melting ice cubes. The only (nonextremal) chances involved are epistemic chances.

In conclusion, both objective and epistemic chance may be objectively informed, and may wear scientific credentials. What distinguishes objective chance are its platitudes. For a probability function to be an objective chance, it must fit the connections to credence, possibility, time, intrinsicness, lawhood,

[38] In this vein, Lewis speaks of 'a kind of counterfeit chance' ([1986], p. 120), invoking the approaches of (Jeffrey [1965]) as well as (Skyrms [1980]). He points out that 'It is a relativized affair, and apt to go indeterminate, hence quite unlike genuine chance' ([1986], p. 120).

and causation. At a deterministic world, the only sort of function that can fit these connections is one that outputs just 0s and 1s. Thus, there cannot be deterministic chance. All there can be is deterministic ignorance.

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